A TEXT-BOOK

ON

ROOFS AND BRIDGES.

PART II.

GRAPHIC STATICS.

BY

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PREFACE.

The course of instruction in roofs and bridges given to students of civil engineering in Lehigh University consists of four parts; first, the computation of stresses in roof trusses and in all the common styles of simple bridge trusses; second, the analysis of stresses by graphic methods; third, the design of a bridge, which includes the proportioning of details and the preparation of working drawings; and fourth, the discussion of cantilever, suspension, continuous and arched bridges. In the following pages the second part of this course is presented, together with additional matter so as to form a tolerably complete treatise on Graphic Statics as applied to the discussion of common roofs and bridges.

In an elementary text-book of this kind it is not expected that much will be found that is new except method of arrangement and presentation. Attention is called, however, to the abbreviated processes employed in some of the diagrams for wind stresses, to the determination of stresses due to initial tension, and to portions of the analysis of maximum moments and shears under locomotive wheel loads as possessing points of novelty and practical value. These new features are due to the experienced Instructor whose name appears on the title page in connection with my own; the larger portion of the text has also been written by him, and the cuts and plates are his work.

The universal approbation expressed concerning the utility of the blank leaves in Part I leads me to insert them in this volume also. On these pages students may record in permanent form the numerical computations which are always requisite preparatory to graphical analysis, and also make free hand sketches of some of the stress diagrams required in the problems. But I regard it as essential that a few well chosen cases shall be thoroughly and completely worked out as indicated on the plates, which show the manner in which for many years I have required students to finish drawings in Graphic Statics. Here, as in Part I, the minimum as well as the maximum stresses are determined for most of the examples, and all varieties of loading are treated so that students may be able to work in accordance with all kinds of specifications.

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LEHIGH UNIVERSITY, BETHLEHEM, PA.,
December 18, 1889.

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GRAPHIC STATICS.

CHAPTER I.

PRINCIPLES AND METHODS.

ART. 1. THE FORCE TRIANGLE.

A force is determined when its magnitude, direction, and line of action are known, and accordingly it may be graphically represented by the length, direction, and position of a straight line. Forces are given in pounds, tons, or kilograms, while the lengths of lines are measured in inches or centimeters. If the scale of force be four tons to an inch, a line 1.32 inches will represent a force of 5.28 tons; thus the magnitude of forces will be directly measured by means of the scales used in draughting.

The resultant of two or more forces is a single force which produces the same effect as the forces themselves, and may therefore replace them. Let two forces P_1 and P_2 which act in the same plane upon the point *m* be represented in magnitude and position by the lines *mn* and *mp*, and in direction by the arrows. Let the parallelogram be completed by drawing a line through *n* parallel to P_2, and a line through *p* parallel to P_1, and then let *m* be joined with their point of intersection. This

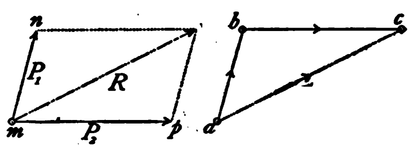


Fig. 1.

line, designated by *R*, represents the resultant of the two given forces. To find the magnitude of this resultant by the analytic method, let θ be the angle included between P_1 and P_2; then from either of the triangles composing the figure a well known theorem of geometry gives

$$R^2 = P_1^2 + P_2^2 + 2P_1P_2 \cos \theta.$$

For instance, if $P_1 = 30$ pounds, $P_2 = 50$ pounds, and $\theta = 75$ degrees, there is found by computation $R = 64.6$ pounds.

The graphic method of finding the resultant consists of the following operations: On a sheet of paper, with the help of a ruler and protractor, from a point m two indefinite lines are drawn, making an angle of 75 degrees with each other. Using a suitable scale, say of 40 pounds to the inch, the distance mn is laid off equal to 30 pounds and mp equal to 50 pounds. With a ruler and triangle the parallelogram is completed, and m joined with the opposite vertex. This line is now measured by the scale, and the value of the resultant is found.

It will be seen that it is not necessary to construct the entire parallelogram, since the triangles on the opposite sides of the diagonal are equal. The triangle above the diagonal can be constructed by drawing a line through n parallel to P_2, laying off upon it the value of P_2, and then joining its end to m; similarly the lower triangle can be independently drawn. Either of these triangles is called the force triangle.

Usually the lines of action of the given forces form part of a diagram upon which it is not desirable to construct the force triangle. In this case let any suitable point a be selected, and ab be drawn parallel and equal to P_1; then through b let bc be drawn parallel and equal to P_2, and let a be joined with c. The line ac represents the magnitude of the resultant R, and is measured by the same scale as that used in laying off ab and bc. The angles bac and bca may be measured by the protractor, and these are the angles which R makes with P_1 and P_2. The

direction in which the resultant acts is indicated by the arrow upon *ac*, and this is seen to be opposed to the directions of those upon *ab* and *bc* in following around the triangle. Finally, the line of action of the resultant R must pass through *m*, the point of application of the given forces P_1 and P_2. Hence, after constructing the force triangle *abc*, the resultant R is found in magnitude, direction, and line of action by drawing through *m* a line equal and parallel to *ac*.

The above operation is termed composition of forces, P_1 and P_2 having been combined into one. The reverse process of decomposition or resolution of forces may also be effected by the force triangle. For instance, let R_1 in Fig. 1 be given, and let it be required to find its components in the directions of *mn* and *mp*. Let *ac* be drawn equal and parallel to R, and through its extremities let *ab* and *cb* be drawn parallel to the given directions; these lines intersect in *b*, and when they are measured by the scale the magnitude of the components will be known. The directions of *ab* and *bc* are opposite to that of *ac* in following around the triangle. Lastly, through *m*, the point of application of R, let P_1 and P_2 be laid off upon the given directions, equal to *ab* and *bc*, and the lines of action of the components are determined.

A number of forces are said to be in equilibrium when no tendency to motion is produced in the body upon which they act. In Fig. 1 suppose a force, P_3, equal and opposite to R, to be applied at *m*; then this force together with P_1 and P_2 will be in equilibrium, for the last two may be replaced by their resultant R, which by the conditions specified holds P_3 in equilibrium. The corresponding force triangle will be *abc* with the direction of *ac* reversed, so that all the forces around the triangle have the same direction; hence, when three forces are in equilibrium, they form a closed force triangle.

When three forces whose lines of action lie in a plane and intersect in one point are in equilibrium, any one may be determined when two are given. In Fig. 2 let P_1 and P_2 be given to find P_3. Let *ab* be laid off equal and parallel to P_1, and from *b* let *bc* be drawn equal and parallel to P_2: then *ca*, the closing side of the triangle, represents P_3 in magnitude and direction. As its line of action must also pass through *m*, the force P_3 is drawn equal and parallel to *ca*, and in the same direction, thus completing the solution.

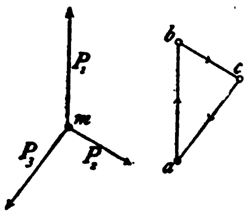


Fig. 2.

Should only one force be given, together with the lines of action of the other two, their magnitudes and directions may be found. Let P_1 and the lines of action of P_2 and P_3 be given. Draw *ab* parallel to P_1, mark off its length according to scale, and through its extremities draw lines parallel to P_2 and P_3; these lines intersect in *c*, and the length of *bc* gives the magnitude of P_2, its direction being from *b* to *c*, while *ca* represents P_3 in the same respects.

The force triangle is the foundation of the science of graphic statics. By it all problems relating to the composition and resolution of forces can be solved, when the forces are but three in number and act in the same plane upon a common point.

Problem 1. Find the magnitude of the resultant of two forces making an angle of 60 degrees with each other, one being 25 pounds and the other 40 pounds.

Prob. 2. The lines of action of two forces, of 50 and 30 pounds respectively, make an angle of 120 degrees. What is the magnitude of the force that holds them in equilibrium and the angles that it makes with each of them?

ART. 2. THE FORCE POLYGON.

When it is required to find the resultant of a number of forces acting in the same plane and having a common point of

application, the resultant of two of the forces may be found by Art. 1, a third force may then be united with it to obtain a second resultant, and this operation continued until all the forces are combined. In Fig. 3, the line R_1 is the resultant of P_1 and P_2, the line R_2 is the resultant of R_1 and P_3, and R is the resultant of R_2 and P_4, and therefore of the given forces P_1, P_2, P_3, and P_4. It is, however, not necessary to construct these resultants in order to find R, if the dotted lines be drawn parallel and equal to P_2, P_3, and P_4.

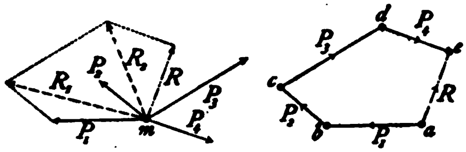


Fig. 3.

The polygon shown by broken lines is called the force polygon; the resultant R forms its closing side, and each of the other sides represents one of the given forces. The diagram *abcde* shows the polygon as it is generally drawn with the diagonals omitted. The direction of the resultant is opposed to the direction of all the given forces in following around the sides of the polygon; thus the arrow on *ae* has the reverse direction of the other arrows.

The force polygon may therefore be constructed as follows:

> Draw in succession lines parallel and equal to the given forces, each line beginning where the preceding one ends, and extending in the same direction as the force it represents. The line joining the initial to the final point represents the resultant in direction and magnitude.

To produce equilibrium with P_1, P_2, P_3, and P_4, a force equal and opposite to R must be applied at *m*. This added force in the force polygon is equal to *ea* with its former direction reversed, and the distance from the initial to the final point in the construction of the polygon becomes zero.

Hence, if a number of forces lying in the same plane and having a common point of application are in equilibrium, they will form a closed force polygon, and in passing around it all the forces will have the same direction.

In either of the above cases it makes no difference in what order the forces are arranged in the force polygon. Thus in Fig. 3 the sides of the force polygon are drawn in the order P_1, P_2, P_3, P_4, R; but the same value of R, both in intensity and direction, will be obtained if they are drawn in any other order, as, for example, P_2, P_1, P_4, P_3, R. Again, in Fig. 4, let the four

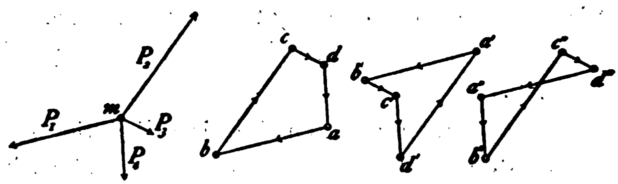


Fig. 4.

forces which meet at m be in equilibrium; then taking them in the order P_1, P_2, P_3, P_4 the force polygon $abcd$ is drawn, in the order P_1, P_3, P_4, P_2 the polygon $a'b'c'd'$ results, and in the order P_4, P_2, P_3, P_1 the polygon $a''b''c''d''$ is found, each of which graphically represents the given forces. In the last case it is seen that two of the lines in the force polygon cross each other; this is of frequent occurrence in practical problems.

The force triangle (Art. 1) is but a particular case of the force polygon, namely, when the forces are but three in number. The word polygon is hence often used in a general sense as including that of the triangle. From three forces in equilibrium two force triangles may be drawn; from four forces in equilibrium six force polygons can be formed.

Prob. 3. Draw a force polygon for five forces in equilibrium, and prove that any diagonal of the polygon is the resultant of the forces on one side and holds in equilibrium those on the other.

Prob. 4. Let $P_1 = 100$ pounds, $P_2 = 175$ pounds, and $P_3 = 60$ pounds, and let the angles which they make with each other be $P_1mP_2 = 135°$, $P_2mP_3 = 87°$, $P_3mP_1 = 138°$. Draw three force polygons and determine from each the value of the resultant, and the angle that it makes with P_1.

ART. 3. CONDITIONS OF EQUILIBRIUM.

When several forces lie in the same plane the necessary and sufficient conditions of static equilibrium are that there shall be no tendency to motion, either of translation or rotation. Analytically this is expressed by saying that the algebraic sum of the components, both horizontal and vertical, of the forces must be zero, and that the algebraic sum of the moments of the forces must also be zero.

When the given forces have a common point of application, the graphic condition for equilibrium is that the force polygon must close. For, if it does not close the line joining the initial with the final point represents the resultant of the given forces (Art. 2), and this resultant will cause motion; and if it does close there exists no resultant. Therefore, if the given forces which meet at a common point are in equilibrium the force polygon must close; and conversely, if the force polygon closes the given forces must be in equilibrium.

When several forces lying in the same plane have different points of application, so that their lines of action do not intersect in the same point, and are in equilibrium the force polygon must also close, since no resultant exists. Thus, suppose the given forces to be four in number, let the directions of two of these be produced until they intersect and their resultant found; this resultant must pass through the point of intersection of the remaining two forces, since equilibrium obtains. Hence the rule above established for forces acting at a common point applies also to this case, and the force polygon must close if they are in equilibrium.

If several forces have different points of application, and the force polygon does not close, the line joining the initial and final points represents the intensity and direction of the resultant. For, as a force can be considered as acting at any point in the line of its direction two of them may be combined into a resultant, and this resultant may be combined with one of the other forces, and so on until the final resultant is obtained in the same manner as in Art. 2.

If several forces have different points of application, and the force polygon closes, it is not necessarily true that the given forces are in equilibrium. For example, let a beam or stick be acted upon by three forces as shown in Fig. 5, the forces P_1, P_2, and P_3 being equal, P_1 and P_2 making an angle of 30 degrees with the horizontal and P_3 being vertical. It is plain that equilibrium is here impossible, and yet the force polygon *abc* closes. Upon reflection it will be seen that the equilibrium of the beam under the action of the three given forces can only be maintained by a couple, that is, by two equal parallel forces acting in opposite directions. It is because the resultant of the forces of a couple is zero that the force polygon closes in this case; and it will be found that in all instances of non-equilibrium where the force polygon closes that a couple is necessary to maintain equilibrium.

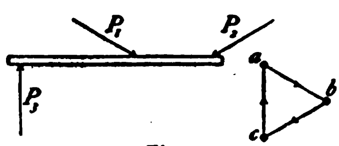


Fig. 5.

The above conditions apply to forces lying in one plane. It is rare in problems relating to roofs and bridges that forces acting in different planes need to be considered, and hence in the following pages it will always be understood, unless otherwise stated, that the forces under discussion lie in the same plane.

Prob. 5. In Fig. 5, let each of the forces be 100 pounds, and let the distance between the points of application of P_1 and P_2 be 4 feet, and between those of P_1 and P_3 be 5 feet. Compute the magnitude of the forces of a horizontal couple to maintain equilibrium when the vertical distance between their points of application is 3 inches. Draw the forces of the couple in both diagrams of Fig. 5.

ART. 4. STRESSES IN A CRANE TRUSS.

As an example of the application of the preceding principles let it be required to graphically determine the stresses in the members B_2, B_3, etc., of the crane truss shown in the left-hand diagram of Fig. 6, due to a load P_1 acting at the peak. The member B_2 is called the tie, B_3 the jib, B_4 the post, and B_5 the back-stay. The post is vertical and its length is 16 feet, the length of the jib is 30 feet, of the tie 21.5 feet, and of the back-stay 20 feet; from these dimensions the diagram of the crane truss is constructed. The load P_1 is taken as 5 tons.

Using a scale of 2 tons to an inch, the construction of the stress diagram is begun by laying off *ab* parallel to P_1 and equal to 5 tons. Now at the peak the force P_1 is resolved into two forces whose lines of action are in the two members B_2 and B_3; hence, by Art. 1, draw *ac* parallel to B_2 and *bc* parallel to B_3, thus obtaining the force triangle *abc*; the length of *ac* gives the stress in B_2, and that of *bc* gives the stress in B_3. Next passing to the apex *n* the stress in B_2 is known and those in B_4 and B_5 are to be found; hence from *a* and *c* draw parallels to these members and these lines, intersecting at *e*, give *ce* as the stress in B_4, and *ae* as that in B_5. This completes the force diagram.

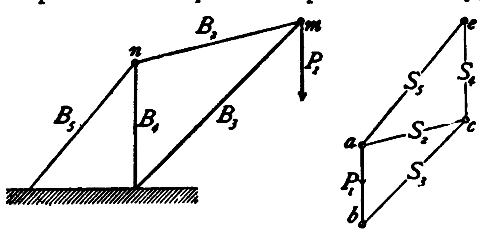


Fig. 6.

The next step is to determine the character of the stresses, that is, whether they are tension or compression. Beginning with the triangle *abc*, which represents the forces acting at the apex *m*, the direction of *ab* is known to be downward, hence following around the triangle (Art. 1) the stress S_3 acts from *b* toward *c*, and S_2 from *c* toward *a*; transferring these directions to the lines of action at the apex *m* it is seen that S_3 acts toward *m*, and is therefore compression, while S_2 acts away from *m*, and is therefore tension. Passing now to the apex *n* the stress S_2 in B_2 is known to be tension and hence it acts away from *n*, accordingly in the force triangle *cae* it acts from *a* toward *c*; hence S_4 acts from *c* to *e*, and S_5 from *e* to *a*; transferring these directions to the lines of action at *n* it is found that S_5 is tension and S_4 is compression.

Applying the scale to the lines of the force diagram the following results are now found, the sign + denoting tension and — denoting compression:

$$S_2 = +6.7, \quad S_3 = -9.35, \quad S_4 = -6.85, \quad S_5 = +10.8 \text{ tons.}$$

These are the stresses in the members B_2, B_3, B_4, and B_5 due to the load of 5 tons acting at the peak. If 10 tons were hung at the peak it is plain that each line of the force triangle would be twice as long as before, or the stresses in the members would be double the values above given.

The two parts of Fig. 6 are called the 'truss diagram' and the 'stress diagram' respectively. Each triangle in the stress diagram *abce* corresponds to the forces acting at one of the apexes in the truss diagram, so that it may be said that the two figures are reciprocal.

Prob. 6. In Fig. 6 let B_4 be vertical and let $B_2 = 30$, $B_3 = 45$, $B_4 = 50$, and $B_5 = 90$ feet. Draw the stress diagram and determine the stresses in all the members due to a force of 6 tons which acts at an angle of 30 degrees to the right of the vertical drawn through the peak *m*.

ART. 5. STRESSES IN A POLYGONAL FRAME.

In Fig. 7 let $B_1B_2B_3B_4$ be a polygonal frame which supports the loads P_3 and P_4 and which is itself suspended by the forces P_1 and P_2 acting in two ropes. The frame being in equilibrium under the action of the exterior forces P_1, P_2, P_3, and P_4, it is required to find the stresses in the members B_1, B_2, B_3, and B_4.

As the exterior forces P_1, P_2, P_3, and P_4 are in equilibrium, the force polygon representing them must close (Art. 3). First, then, let the force polygon *abcd* be drawn, *ab* representing P_1

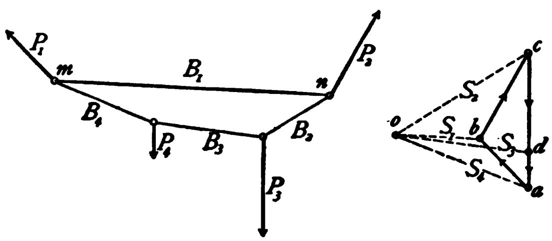


Fig. 7.

in magnitude and direction, *bc* representing P_2, and so on. Now at each apex of the polygonal frame there are three forces which are in equilibrium. Thus at *m* the force P_1 is known, and if from *b* and *a* lines be drawn parallel to B_1 and B_4 these intersect in *o*, giving *bo* as the stress S_1 in B_1 and *ao* as the stress S_4 in B_4. Similarly at each of the other apexes the exterior force may be resolved into components in the two given directions. Thus S_1, S_2, S_3, and S_4 are found as the stresses in B_1, B_2, B_3, and B_4.

To find the character of these stresses it is only necessary to follow around the sides of each force triangle in the direction indicated by the given force and then to transfer these directions to the corresponding apex of the frame. Thus, at the apex *n* the direction of P_2 is known and the corresponding force triangle is *bco*; in this P_2 acts from *b* toward *c*, hence S_2 acts from *c* toward *o* and S_1 acts from *o* toward *b*; transferring these directions to *n* it is found that S_1 acts toward and S_2 away from *n*, thus showing S_1 to be compression and S_2 to be tension. In like manner it is found that S_3 and S_4 are also tension.

Prob. 7. If the frame in Fig. 7 be inverted, draw the force diagram and determine the character of the stress in each member.

Prob. 8. In Fig. 7 let each of the forces P_1, P_2, P_3, and P_4 be vertical and equal to 100 pounds. Let B_1 and B_2 be horizontal, the length of the former being 6 feet and that of the latter being 2 feet. Let the lengths of B_3 and B_4 be 5 feet. Draw the force diagram and find the magnitude and character of the stress in each member.

ART. 6. THE EQUILIBRIUM POLYGON.

When a number of forces acting upon a body do not meet in a common point the magnitude and direction of their resultant is found by the closing line of the force polygon (Art. 3), but its line of action is not determined. This will now be found by means of another diagram which is called the equilibrium polygon.

Let four forces represented by P_1, P_2, P_3, and P_4 be given in magnitude, direction, and line of action, and let it be required to fully determine their resultant. Constructing the force polygon *abcde*, the length of the closing line *ea* represents the magnitude of the resultant, and its direction is from *a* toward *e*, being opposite to those of the other forces in following around the polygon (Arts. 2 and 3). Now select any point *o* and draw the lines *oa*, *ob*, *oc*, *od*, and *oe* to the vertices of the force polygon, thus forming five force triangles. In the force triangle *oab* the lines *oa* and *ob* represent two forces which can hold *ab* in equilibrium if their directions be from *b* to *o* and from *o* to *a*. Thus each of the forces in the force polygon can be replaced

by its components shown by the broken lines; for example, P_2 has the components S_1 and S_2. Now through any point *m* on the line of action of P_1 draw the lines B_1 and B_5 parallel to S_1 and S_5 respectively, and let B_1 intersect the line of action of the force P_2 at *n*. Through *n* draw B_2 parallel to S_2, and so on in succession, until finally B_4 is drawn parallel to S_4. The lines B_4 and B_5 will intersect at some point *r*; through this point draw a line *R* parallel to *ae* and the line of action of the resultant is determined. For, by the construction the forces S_4 and S_5 are the components of the resultant *R* or *ae*, and as their lines of action are in B_4 and B_5 the resultant must pass through the point where they intersect.

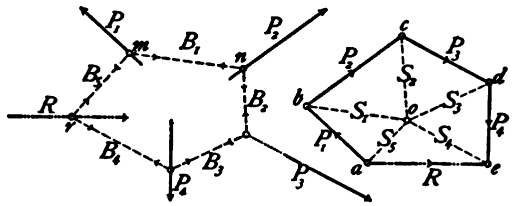


Fig. 8.

If in Fig. 8 there be applied at the point *r* a force P_5 equal to *R* but opposite in direction the forces P_1, P_2, P_3, P_4, and P_5 are in equilibrium and the force polygon closes. The polygonal frame $B_1B_2B_3B_4B_5$ thus holds the given forces in equilibrium by means of the stresses of tension and compression acting in its members. For the case shown in the figure these stresses are all tensile, and their values are given by the lines S_1, S_2, etc., in the force polygon. The lines of this frame are hence called an 'equilibrium polygon.' The polygonal frame in Fig. 7 is an equilibrium polygon which holds the exterior forces in balance.

The graphic condition of equilibrium for several forces not meeting at the same point may now be expressed by saying that both the force polygon and the equilibrium polygon must close. If the former closes and the latter does not the given forces are not a system in equilibrium. For example, the three forces P_1, P_2, and P_3 in Fig. 9 are equal in magnitude and make angles of 120 degrees with each other, but they are not in equilibrium because their lines of action do not intersect in the same point. The force polygon *abc* here closes. Select any point *o* and draw the lines *oa*, *ob*, and *oc*, thus resolving the force P_1 into the components S_1 and S_3, the force P_2 into S_1 and S_2, and the force P_3 into S_2 and S_3. Now select any point *m* on the line of action of P_1 and draw *mr* and *mn* parallel to S_1 and S_3; from *n*, where *mn* intersects the line of action of P_3, draw *nr* parallel to S_2. Then *mr* and *nr* intersect at *r* which is not on the line of action of P_2, and hence the three given forces cannot be held in equilibrium by an equilibrium polygon. In this case it is said that the equilibrium polygon does not close. If, however, the force P_2 be moved parallel to itself until its line of action passes through *r* the force polygon closes and the forces will be in equilibrium.

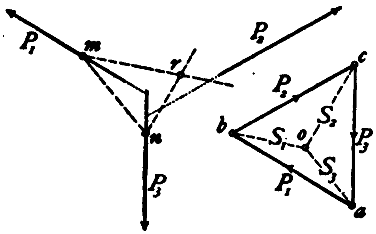


Fig. 9.

The point *o* in the plane of the force polygon is called 'the pole,' and the lines *oa*, *ob*, etc., are sometimes called 'rays.' Since the position of the pole may be selected at pleasure it follows that for any given system of forces an infinite number of equilibrium polygons can be constructed. The pole *o* may be taken either within or without the force polygon as may be most convenient for the solution of the problem under consideration.

Prob. 9. Given two forces of 100 and 180 pounds acting at an angle of 5 degrees with each other, the point of intersection not being within the limits of the drawing. Find the magnitude and direction of the resultant by the force polygon, and its line of action by the equilibrium polygon.

ART. 7. PROPERTIES OF THE EQUILIBRIUM POLYGON.

Let Fig. 10 represent a system of forces P_1, P_2, . . . P_6 held in equilibrium by the jointed frame or equilibrium polygon whose members are B_1, B_2, . . . B_6. This is constructed by first

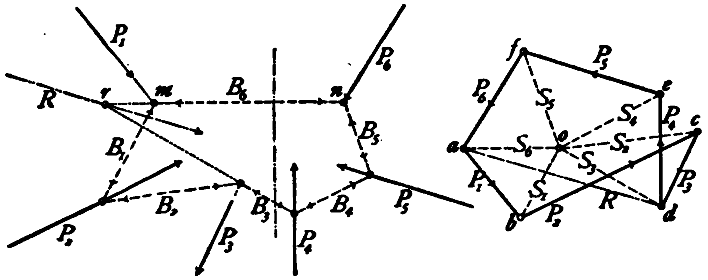


Fig. 10.

drawing the force polygon *abcdef* which must close, then selecting a pole *o* and drawing the rays *oa, ob,* . . . *of*, to which the members of the equilibrium polygon are made respectively parallel. The character of the stresses in these members is determined from the force polygon; thus in the triangle *abo* the directions of *bo* and *oa* must be from *b* to *o* and from *o* to *a* in order to maintain equilibrium; transferring these directions to the other diagram, it is seen that the stresses in B_6 and B_1 act toward the apex *m*, and hence are compression. Passing next to the vertex *n* the stress in B_5 is also found to be in compression, and so on. (Art. 4.)

Let this equilibrium polygon be cut by a section shown by the broken and dotted line; the stresses in B_3 and B_6, the members cut, are given by S_3 and S_6 in the force diagram, and form the closing sides of the polygon *abcdo* and also of the polygon *defao*. That is, the stresses in B_3 and B_6 hold in equilibrium the external forces P_1, P_2, and P_3, and also the external forces P_4, P_5, and P_6. Therefore the following principle is established:

The internal stresses in any section hold in equilibrium the external forces on either side of that section.

This principle, it will be observed, is the same as that applicable to the internal stresses in a beam (Mechanics of Materials, Art. 15) or to the internal stresses in a truss (Roofs and Bridges, Part I, Art. 4).

Since the stresses in B_3 and B_6 hold in equilibrium the external forces P_1, P_2, and P_3 the resultant of the former must be equal and opposite to the resultant of the latter. This is also seen in the force polygon where the line *ad* gives the magnitude of this resultant. To find its line of action let B_3 and B_6 be produced until they meet in *r*, and through *r* draw a line equal and parallel to *ad*. Thus results the following important principle:

> The resultant of the external forces on the left of any section passes through the intersection of the sides of the equilibrium polygon cut by that section, its magnitude and direction being given by the force polygon.

The resultant of the external forces on the right of the section is the same in magnitude and line of action as that of those on the left, but its direction is reversed.

When a system of parallel forces are in equilibrium, the force polygon becomes a straight line and the equilibrium polygon has an important special property which will now be deduced. Let P_1, P_2, and P_3 in Fig. 11 be three downward forces, held in equilibrium by the two upward forces P_4 and P_5; for example, the former might be loads acting on a beam and the latter the reactions of the supports. The force polygon here is *abcdea*, the lines *ab*, *bc*, and *cd* being laid off downward while *de* and *ea* are laid off upward, closing the polygon. Selecting any pole *o*, and drawing the rays *oa*, *ob*, etc., the equilibrium polygon B_1B_2 $B_3B_4B_5$ is formed (Art. 6). Now let any two sides B_1 and B_4 be cut by a vertical plane, and let the ordinate intercepted between them be called *y*. The intersection of these sides produced gives the point of application of the resultant of the ex-

ternal forces P_1 and P_2, whose value is given by *eb* in the force polygon; let the horizontal distance from y to this point be called r, and the resultant be called R. The bending moment

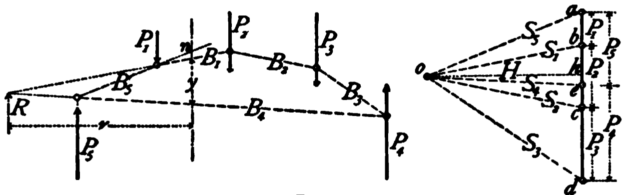


Fig. 11.

M in the given section is then equal to the moment of the resultant of all the forces on the left of that section, or

$$M = Rr.$$

Now in the force polygon let the line *oh* be drawn horizontally through the pole and its value be called H. Then since the triangle *obe* is similar to the triangle which has the base y and the altitude r, and since *eb* is equal to R, we have

$$r : y :: H : R \qquad \text{or} \qquad Rr = Hy.$$

Therefore the bending moment of the external forces on the left of the section is

$$M = Hy.$$

The force H is seen to be the horizontal component of each of the stresses *oa*, *ob*, etc., in the force polygon, that is, of the stresses in the members B_1, B_2, etc., in the equilibrium polygon; it is called the 'pole distance,' and is measured by the same scale of force as the other lines in the force polygon. The following theorem can hence be stated:

> If a structure be subject to parallel forces, the bending moment in any section parallel to the forces is equal to the ordinate y in the equilibrium polygon multiplied by the pole distance H in the force polygon.

Hence by adopting suitable scales the values of the bending moments can easily be found from the diagram. For instance, if the linear scale used in laying off the positions of the external forces be 20 feet to the inch, and if the pole be so selected that the distance H is 5 tons, then the moments are measured by a scale of 5 tons × 20 feet = 100 ton-feet to the inch.

The following is another proof of this important theorem: Let the line B_6 in Fig. 11 be produced until it meets the section in n; let the vertical distance between n and B_6 be called y_6, and that between n and B_1 be called y_1. Let the lever arms of P_1 and P_6 with respect to the section be called p_1 and p_6. The bending moment for the section then is

$$M = P_6 p_6 - P_1 p_1 .$$

But the triangle whose base is y_6 and altitude p_6 is similar to *oae*, and the triangle whose base is y_1 and altitude p_1 is similar to *oab*. Hence $P_6 p_6 = Hy_6$ and $P_1 p_1 = Hy_1$; and accordingly

$$M = H(y_6 - y_1).$$

But since $y_6 - y_1 = y$, this gives $M = Hy$, which is the same relation as before deduced.

Prob. 10. Draw an equilibrium polygon for the five vertical forces given in Fig. 11, taking the pole on the right-hand side of the force polygon.

Prob. 11. Given two parallel forces 12 feet apart and acting in opposite directions, one being 6 tons and the other 2 tons. Find by the force and equilibrium polygons the magnitude and line of action of their resultant.

ART. 8. REACTIONS OF BEAMS.

By the use of the force and equilibrium polygons the reactions of the two supports of a beam carrying given loads may be graphically determined. For example, let the beam in Fig. 12 be subject to two concentrated loads as shown, and be in equilibrium under the action of these loads and the two reactions. If the values of the reactions were known an equilibrium poly-

gon could then be constructed which should act instead of the beam to maintain this equilibrium (Art. 7). But since the loads and reactions constitute a system of forces in equilibrium, the principle that the equilibrium polygon must close (Art. 6) furnishes the means of determining the unknown reactions.

Let P_1 and P_2 be the given loads, and let *ab* and *bc* be drawn equal and parallel to them. Since the force polygon must close the line *ca* then represents the sum of the two reactions.

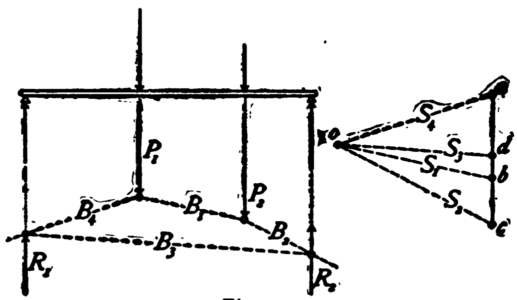


Fig. 12.

Next let any pole *o* be selected, and the rays *oa*, *ob*, and *oc* be drawn, and parallel to these let B_4, B_1, and B_2 be drawn, thus forming part of the equilibrium polygon. This polygon can now be closed by drawing B_3, joining the points where B_4 and B_2 intersect the lines of action of the reactions. Finally, in the force polygon let *od* be drawn through *o* parallel to B_3; thus determining the point *d*; then *cd* and *da* are the two reactions, the former being R_2, and the latter R_1. For, *cd* is the force that holds in equilibrium the stresses S_2 and S_3 in the members B_2 and B_3, and *da* is the force that holds in equilibrium the stresses S_3 and S_4 in B_3 and B_4.

As a second example, let it be required to determine the reactions for the overhanging beam shown in Fig. 13 due to the two given loads. Laying off *ab* and *bc* as before, the sum of the reactions is shown by the line *ca*. Choosing a pole *o* and drawing *oa*, *ob*, and *oc*, the equilibrium polygon is constructed by taking any point on the line of action of P_1 and drawing B_4 and B_1 parallel to *oa* and *ob* respectively. Then from the point where B_1 intersects the line of action of P_2 the line B_2 is drawn parallel to oc. Now B_4 intersects the line of action of R_1 at m and B_2 intersects that of R_2 at n; joining mn by the line B_5 closes the equilibrium polygon, and parallel to this closing line od is to be drawn in the force diagram, thus determining the two reactions cd and da:

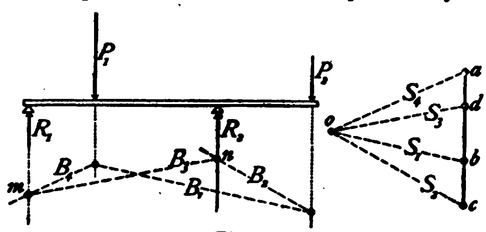


Fig. 13.

In both the above figures, any ordinate drawn in the equilibrium polygon gives the bending moment in the beam at the point vertically above it (Art. 7). In Fig. 13 where the sides of the equilibrium polygon cross there is no ordinate, and this corresponds to the position of the inflection point in the beam where the horizontal stresses change from tension to compression.

Prob. 12. Given an overhanging beam as in Fig. 13, its length being 18 feet, and the distance between the supports 14 feet. Determine the reactions due to three loads, one of 200 pounds at 3 feet from the left end, one of 80 pounds at 6 feet from the left end, and one of 90 pounds at the right end.

ART. 9. SIMPLE BEAMS UNDER CONCENTRATED LOADS.

By applying the principles of the preceding articles the vertical shears and the bending moments may be found for all sections of a beam having only two supports and subject to any number of concentrated loads. For example, consider a simple beam 20 feet long, carrying five loads whose positions, and weights in pounds, are shown in Fig. 14. The reactions of the supports are found by laying off the loads successively on the force polygon, or load line af, the first load being ab, the second bc, etc. Select the pole o, draw the rays from o, and construct the equilibrium polygon m, n, etc., its closing line

being *ms*. Then through *o* draw *og* parallel to *ms*, and the lines *fg* and *ga* will represent the reactions of the right and left supports.

Between the left support and the first load the vertical shear equals the reaction *ga*; between the first and second loads the vertical shear is $ga - ab = gb$; between the second and third

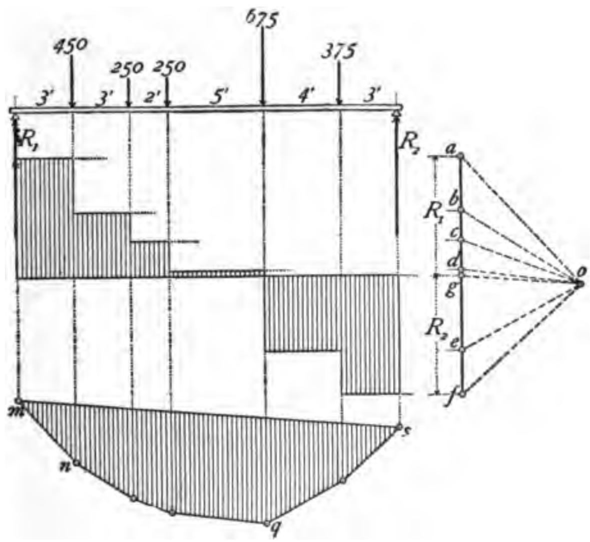


Fig. 14.

loads it is $ga - ab - bc = gc$; and so on. At the fourth load the shear changes from positive to negative, and at the right support its value is the reaction *gf*. The diagram shown in the figure above the equilibrium polygon gives these shears, and the manner of its construction is apparent, each step being one of the loads. This is called the shear diagram.

The ordinates in the equilibrium polygon, or moment diagram, give the bending moments in the corresponding sections of the beam. It is seen that the maximum ordinate is where the sides of the equilibrium polygon meet that are parallel to *od* and *oe*, the rays on opposite sides of and adjacent to *og*, the line parallel to the closing side *ms* of the equilibrium polygon. As the shear immediately on the left of the corresponding section is positive, being measured from *g* to *d*, and that on the right is negative, being measured from *g* to *e*, the important relation is obtained that the maximum bending moment occurs at the section where the vertical shear passes through zero.

In making the actual construction for Fig. 14 the linear scale used in laying off the beam and the positions of the load was 5 feet to an inch, and the force scale used in the force polygon was 800 pounds to an inch. The pole distance was taken as 1 000 pounds, and hence the moment scale was 5 000 pound-feet to an inch. Any ordinate in the shear diagram, measured by the force scale, gives the vertical shear in pounds; thus, between the second and third loads the shear is + 300 pounds, and between the fourth and fifth loads it is − 625 pounds. Any ordinate in the moment diagram, measured by the moment scale, gives the bending moment in pound-feet; thus, the maximum bending moment is 5 500 pound-feet. Fig. 14, however, as here printed, is about one-half the size of the actual construction.

Prob. 13. Construct the shear diagram and moment diagram for a beam 16 feet long, carrying two loads, each of 4 000 pounds, one being at 5 feet from the left end, and the other at 5 feet from the right end.

ART. 10. SIMPLE BEAMS UNDER UNIFORM LOADS.

Let a simple beam whose span is l be uniformly loaded with the weight w per linear unit; then each reaction is equal to half the total load or $\frac{1}{2}wl$. The load may be represented graphically by the shaded rectangle on the beam whose base is l and altitude w.

For any section at a distance x from the left support the vertical shear is $V = \frac{1}{2}wl - wx = w(\frac{1}{2}l - x)$; if V be an ordi-

nate corresponding to an abscissa x this is the equation of a straight line. Thus when $x = 0$, $V = \frac{1}{2}wl$; when $x = \frac{1}{2}l$, $V = 0$; and when $x = l$, $V = -\frac{1}{2}wl$. The shear diagram is hence constructed by laying off *gi* equal to the span, making *gf* and *ik* equal to $\frac{1}{2}wl$ and joining *f* with *k*.

The bending moment in a section distant x from the left support is $M = \frac{1}{2}wl \, . \, x - wx \, . \, \frac{1}{2}x = \frac{1}{2}w(lx - x^2)$. This is the equation of a parabola; for $x = 0$ and $x = l$ the value of M is 0; for $x = \frac{1}{2}l$, M reaches its maximum value $\frac{1}{8}wl^2$. The moment diagram may hence be constructed by laying off *mn* equal to the span, drawing *qr* at the middle equal to the maximum moment, and then constructing the parabola

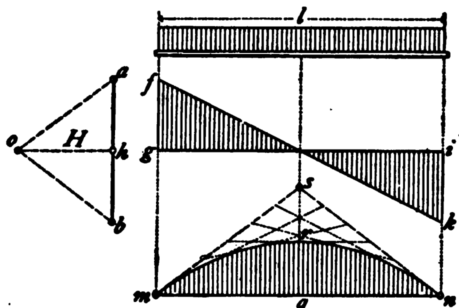


Fig. 15.

mrn. To do this the lines *ms* and *ns* are drawn, *rs* being made equal to *qr*, these are divided into the same number of equal parts and the points of division joined as shown, thus determining tangents to the parabola.

If the entire load on the beam were concentrated at the middle, *aba* would be the force polygon, and *bh* and *ha* the two reactions. Now let *o* be a pole having the pole distance H, and let the equilibrium polygon *msn* be constructed. Then from the similar triangles *oah* and *msq*,

$$H : \tfrac{1}{2}wl :: \tfrac{1}{2}l : qs.$$

Hence if H be equal to unity on the scale of force, the ordinate *qs* has the value $\frac{1}{4}wl^2$, and since *qr* is $\frac{1}{8}wl^2$ the maximum moment for a single concentrated load at the middle is twice as great as that due to the same load when uniformly distributed.

Prob. 14. Prove that the sum of all the moments due to a uniform load is two-thirds of the sum of all the moments due to the same load when concentrated at the middle.

ART. 11. OVERHANGING BEAMS.

Let a beam be taken with one overhanging end and bearing a number of concentrated loads as shown in Fig. 16. The loads are given in pounds and the distances in feet. If in the force polygon the loads be laid off successively in the order in

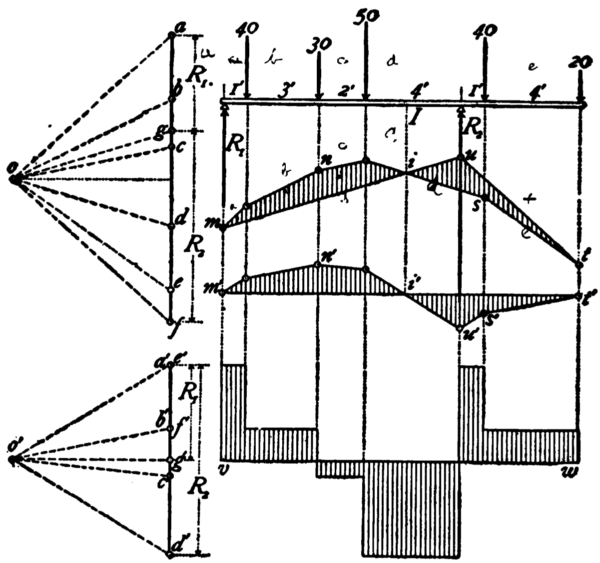


Fig. 16.

which they are on the beam, and the equilibrium polygon *m* . . *n* . . *stu* be constructed, the ray *og* drawn parallel to the closing line *mn* will determine the reactions *fg* and *ga*. The sides of the equilibrium polygon are found to cross each other at *i*, and the ordinates to the right of this point lie on the opposite side of the closing line from those on the left. The ordinates on the left being regarded as positive, those on the right are negative, and they give the bending moments for all

sections in the beam. The point I, where the bending moment is zero, is called the inflection point; on the left of this point the lower fibers are in tension while on the right they are in compression.

In order to construct a moment diagram whose ordinates shall be measured from a horizontal line $m't'$, the numerical values of the reactions should first be computed; these are $R_1 = 60$ pounds and $R_2 = 120$ pounds. Now form a force polygon, or load-line, for the reactions and loads by laying off the reaction R_1 from g' upward to a', then the first three loads in succession from a' downward to d', then the reaction R_2 from d' upward to e', and finally the remaining loads from e' downward to g'. Take the pole o' on a perpendicular to the load line at g'. The equilibrium polygon $m' \ldots n' \ldots u's't'$ can then be constructed, each of whose ordinates is equal to that in the polygon $m \ldots n \ldots stu$.

The method of constructing the shear diagram on the axis vw will be understood without further explanation than that given in Art. 9. It is seen that the shear passes through zero at two points, one where the maximum positive moment occurs, and the other at the right support where the negative moment is a maximum.

The linear scale used in the actual construction of Fig. 16 was 4 feet to an inch, and the force scale was 60 pounds to an inch; the pole distance being 100 pounds, the moment scale was 400 pound-feet to the inch. In the figure as printed the scales are one-half these values. By measurement it is found that the maximum shear is 60 pounds, the maximum positive moment 120 pound-feet, and the maximum negative moment 140 pound-feet.

For the case of a uniform load a shear diagram and moment diagram may be constructed by computing the maximum ordinates and then drawing the straight lines and parabolas (Mechanics of Materials, Chap. IV). Thus, let Fig. 17 represent a beam 28 feet long with ends overhanging 4 and 6 feet, and let the uniform load be 40 pounds per linear foot. The left reaction is found by computation to be 497.8, and the right reaction to be 622.2 pounds; these might also be obtained graphically by the equilibrium polygon, regarding the load on each portion of the beam as concentrated at its center. The shear is then found to be zero at *c*, distant 8.45 feet from the left support, and at this point the positive moment is a maximum, its value by computation being 1 106 pound-feet. At the left support the negative moment is found to be 320 and at the right support 720 pound-feet. These moments being laid off by scale the curves can be constructed by the method given in Art. 10, it being known that the end parabolas have their vertices at *m* and *q*, and that the middle parabola has its vertex at *n*. The inflection points are equally distant from the place of maximum positive moment, this distance being 7.45 feet in Fig. 17. The diagrams thus furnish full information regarding the distribution of the shears and moments in the beam.

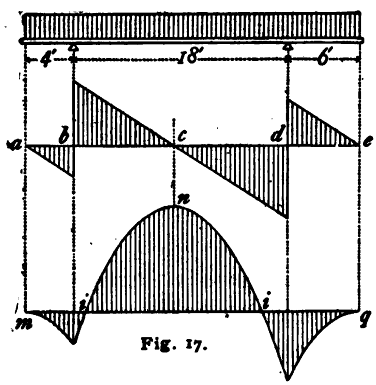


Fig. 17.

Prob. 15. A beam 20 feet long has two overhanging ends, each 5 feet long. Draw the shear and moment diagrams due to a load of 6 tons at one end and a load of 4 tons at the other end.

Prob. 16. A beam 20 feet long has two overhanging ends, each 5 feet long. Draw the shear and moment diagrams due to three loads, one of 6 tons at one end, one of 4 tons at the other end, and one of 8 tons at the middle.

ART. 12. CENTER OF GRAVITY OF CROSS-SECTIONS.

In problems relating to the strength of beams it is necessary to find the position of the neutral axis of the cross-section (Mechanics of Materials, Chap. III). The neutral axis passes through the center of gravity of the cross-section, and in finding the position of the center of gravity the equilibrium polygon may be employed. For instance, let the cross-section of a deck beam shown in Fig. 18 be taken. As this cross-section has an axis of symmetry *AD*, the center of gravity lies on this axis. The area is divided into simple geometrical figures or narrow strips by lines perpendicular to the axis, and at the centers of gravity of these parts forces proportional to their areas are applied. The force and equilibrium polygons are constructed in the usual manner, taking the pole opposite the center *h* of the force line and making the pole distance *oh* equal to *ah*. The extreme sides of the equilibrium polygon *m* . . *n* . . *q* are produced until they meet at *r*, the special position of the pole causing them to form the best intersection, that is, at right angles. According to Art. 6, the point *r* is in the line of action of the resultant of the forces considered, hence *rC* drawn parallel to the forces is the line of action of the resultant. The center of gravity lies on this line and therefore at its intersection with the axis *AD*.

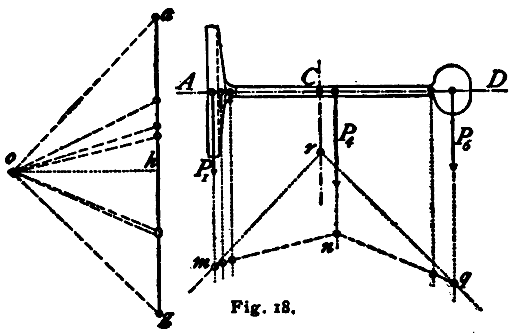


Fig. 18.

If the surface is very irregular in outline it should be divided into strips so narrow that the area of each one is equal to the product of its mean length by its width without appreciable error. If there be no axis of symmetry, the process described above must be repeated for a direction at right angles to the first, and the center of gravity will lie at the intersection of the two resultants found.

Prob. 17. Find the center of gravity of the cross-section of the channel-iron whose dimensions in inches are given in Fig. 19.

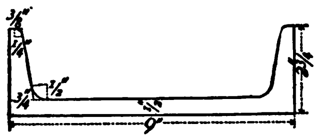


Fig. 19.

Prob. 18. Five parallel forces act in the same direction. Beginning at the left their magnitudes are 45, 120, 30, 225, and 80 pounds, and the successive distances between them are 6, 10, 3, and 8 feet. Determine the magnitude and position of their resultant.

ART. 13. MOMENT OF INERTIA OF CROSS-SECTIONS.

For beams under flexure the bending moment M for any section equals the resisting moment $\frac{SI}{c}$ with reference to the neutral axis in that section in which S is the unit-stress in the most remote fiber distant c from the axis and I is the moment of inertia of the cross-section with reference to the same axis (Mechanics of Materials, Art. 19).

In Art. 12 the method is given for finding the center of gravity through which the neutral axis passes, and the moment of inertia I may be obtained from the same construction by the following rule: Let A be the area of the given cross-section, and A' the area included between the equilibrium polygon and the two lines whose intersection determines the center of gravity; then I is the product of A by A', or $I = AA'$, provided that the pole distance is $\frac{1}{2}A$. This rule will now be demonstrated in connection with a practical example.

Let it be required to find I for the Pencoyd T iron, No. 109,

shown in Fig. 20. The cross-section A measures 4.04 square inches, and by the force and equilibrium polygon the neutral axis EC is found to be 1.32 inches from the base of the T. Now produce sq until it meets the axis at t. The triangles qtu and ofe are similar, as their sides are mutually parallel. Let y be the distance from q to the axis; then

$$tu : y :: ef : oh.$$

But ef equals the area P_6 laid off to scale, and the pole distance oh was made equal to $\frac{1}{2}A$; hence,

$$tu \,.\, \tfrac{1}{2}A = P_6 y.$$

Multiplying this by y, and remembering that $\frac{1}{2}tu \,.\, y$ is the area of the triangle qtu, gives,

$$\text{area } qtu = \frac{P_6 y^2}{A}.$$

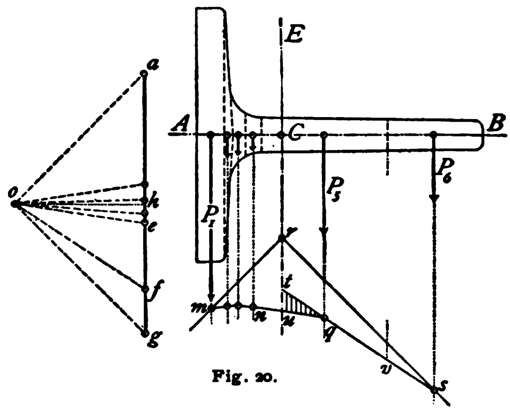


Fig. 20.

The other triangles composing the area between the equilibrium polygon and the lines mr and sr may be expressed in a similar manner. If the area P_6 were of the width dy its moment of inertia would be $P_6 y^2$, and by adding all the areas together there would be found,

$$A' = \frac{I}{A}, \qquad \text{or} \qquad I = AA',$$

in which case the broken line $m \ldots nuqs$ becomes a curve which is tangent to the lines mr and sr at the extreme limits of the given cross-section. This curve may be drawn and the area A' determined either by dividing it into strips or by the planimeter.

By performing the above operations on a full scale drawing for the T iron shown in Fig. 20 the area A' was found by the planimeter to be 1.80 square inches, hence the moment of inertia is

$$I = 4.04 \times 1.80 = 7.27 \text{ inches}^4.$$

This agrees very well with the value given in the manufacturer's pocket-book, which is 7.26 inches⁴.

Prob. 19. Determine the moment of inertia of the channel iron shown in Fig. 19 with respect to an axis through the center of gravity and normal to the web. Determine also the moment of inertia for an axis through the center of gravity and parallel to the web.

ART. 14. GRAPHICAL ARITHMETIC.

In the various operations that are performed on the drawing board cases occur where quantities are to be added, subtracted, multiplied, or divided, and it frequently happens that this may be done by some simple graphical procedure instead of by the common methods of arithmetic. Whatever be the nature of these quantities they are represented by straight lines, the number of units in the length of a line denoting the magnitude of the quantity.

To add together several lines it is only necessary to place them end to end and then measure the total length by the scale.

To subtract one line from another the first is to be laid off from the end of the second in the opposite direction and then their difference can be measured by the scale.

To add algebraically several quantities, some positive and some negative, the sum of the lines representing the negative ones is to be found and subtracted from the sum of the lines representing the positive ones.

To multiply any line of length a by any quantity represented by a line of length m, let OX and OY be drawn making a convenient angle with each other. Lay off Ob equal to unity, Oa equal to a, and Om equal to m. Then Oy is the required product $a \times m$. For, by similar triangles,

$$Oy : Om :: Oa : Ob,$$

whence $$Oy = Oa \times Om = a \times m.$$

By successive applications of this principle the product of several factors may be found.

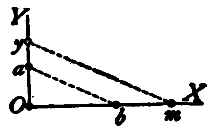


Fig. 21.

To divide a line of length m by one of length b, let Oa be laid off equal to unity, Om equal to m, and Ob equal to b; then making the construction as before, the similar triangles give

$$Oy = Om \div Ob = m \div b,$$

and hence Oy is the quotient found by dividing m by b.

By the help of the principle of similar triangles a number of graphical constructions can be made for adding and multiplying fractions. For instance, let it be required to find the product of the two fractions $\frac{6}{13}$ and $\frac{9}{8}$. On any line OX lay off, as in Fig. 22, $Oa = 13$ and $Oc = 8$; erect the ordinates $ab = 6$ and $cd = 9$; join Ob and Od. Select any abscissa Ox, in this case taken as 10, draw the ordinate xy, and taking Ox' equal to xy, draw the ordinate $x'y'$. Then the product of the two fractions is one-tenth of $x'y'$. For, by similar triangles,

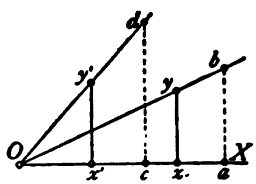


Fig. 22.

$$\frac{ab}{Oa} = \frac{xy}{Ox} \quad \text{and} \quad \frac{cd}{Oc} = \frac{x'y'}{Ox'}.$$

Multiplying these together, term by term, and inserting the values, there results

$$\frac{6}{13} \times \frac{9}{8} = \frac{x'y'}{10}.$$

Measuring $x'y'$ by the scale it is found to be 5.2; hence the required product is 0.52.

Prob. 20. Show by Fig. 21 how to graphically transform the fraction $\frac{2.5}{5.5}$ to an equivalent fraction having 9 for its denominator.

Prob. 21. Make the following graphical constructions: (*a*) to square a given number, (*b*) to extract the square root, (*c*) to find the reciprocal, the final result in each case to be represented by the length of a straight line.

CHAPTER II.

ROOF TRUSSES.

ART. 15. DEFINITIONS AND PRINCIPLES.

A roof truss is a structure whose plane is vertical, and is supported at its ends by the side walls of the building, being so arranged that its principal members are subject only to tensile or compressive stresses under the influence of the loads which it is designed to carry.

The points where the center lines of the adjacent members meet are the centers of the connections which form the joints. The joints of the truss are supposed to be perfectly flexible, and the external forces, consisting of the loads and reactions, to be applied only at the joints.

For stability the elementary figures composing a truss must be triangles, since a triangle is the only polygon which cannot change its shape without altering the lengths of its sides when loaded at one or more joints.

The 'span' of a truss is the distance between the end joints or the centers of the supports, and the 'rise' is the distance from the highest point, or peak, to the line on which the span is measured.

The 'upper chord' consists of the upper line of members extending from one support to the other. Each half of the upper chord of a triangular truss is sometimes designated the 'main rafter.' The lower line of members is known as the 'lower chord' or 'tie rod.'

The 'web members' or 'braces' connect the joints of the upper with those of the lower chord, and may be either verticals, diagonals, or radials.

A member which takes compression is called a 'strut,' and one that takes tension a 'tie.' The upper chord and some of the braces are subject to compression while the lower chord and the rest of the braces are in tension.

The fundamental principles of Graphic Statics as given in Chapter I apply to the determination of the stresses in trusses under given conditions of loading, in the manner indicated in Art. 4. The notation employed in this chapter for designating truss members and loads differs from that previously used, the letters being placed upon spaces instead of upon lines, and any member is named by the letters between which it is situated. Thus, in Fig. 23, AC and BC are the two rafters and CD is the tie rod, while AB designates the load at the peak.

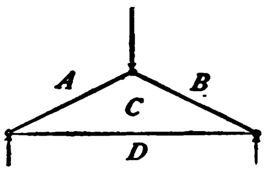


Fig. 23.

Prob. 22. The span in the simple triangular roof truss of Fig. 23 is 26 feet, and the rise is 6 feet 6 inches. Find the stresses in the members due to a load at the peak of 2 050 pounds.

ART. 16. DEAD AND SNOW LOADS.

Four kinds of loads are to be considered in discussing a truss: the weight of the truss itself, the weight of roof covering, the snow, and the wind.

The weight of the truss depends upon its span and rise, the distance between adjacent trusses, the kind of material used in the construction of the roof covering and the truss, and other elements of design. This weight is ascertained from the results of experience. In RICKER'S 'Construction of Trussed Roofs,' page 46, is a table derived from data given by different authori-

ties which seems to afford the best figures now attainable. The following formulas give results approximately agreeing with those found by the use of this table:

For wooden trusses, $W = \frac{1}{2}al(1 + \frac{1}{10}l)$,
For wrought-iron trusses, $W = \frac{3}{4}al(1 + \frac{1}{10}l)$,

in which l is the span in feet, a the distance in feet between adjacent trusses, and W the approximate weight of one truss in pounds. The wooden trusses are to have wrought iron tension members in accordance with the usual practice. It is seen that they are one-third lighter than the wrought iron trusses.

The roof covering consists of the exterior 'shingling' of tin, slate, tiles, corrugated iron, or wooden shingles, resting usually upon timber 'sheathing,' which is supported by 'purlins,' or beams running longitudinally between the trusses and fastened to them at the upper joints. In large roofs the sheathing is laid upon 'rafters' parallel to the upper chord, the rafters resting upon the purlins. The actual weight of the roof covering, rafters, and purlins is to be determined only by computation for each particular case, but the following values will serve for preliminary designs and approximate computations. The weights given are in all cases per square foot of roof surface.

For shingling—tin, 1 pound; wooden shingles, 2 or 3 pounds; iron, 1 to 3 pounds; slates, 10 pounds; tiles, 12 to 25 pounds.
For sheathing—boards 1 inch thick, 3 to 5 pounds.
For rafters—1.5 to 3 pounds.
For purlins—wood, 1 to 3 pounds; iron, 2 to 4 pounds.
Total roof covering—from 5 to 35 pounds per square foot of roof surface.

The snow load varies with the latitude, being about 30 pounds per horizontal square foot in Northern New England, Canada, and Minnesota, about 20 pounds in the latitude of New York City and Chicago, about 10 pounds in the latitude of Baltimore and Cincinnati, and rapidly diminishes southward. On roofs having an inclination to the horizontal of 60 degrees or more this load may be neglected, as it might be expected that the snow would slide off.

The weight of a roof truss with that of the roof covering which it bears is termed the 'dead load' or 'permanent load.'

For the purpose of securing uniformity in the solution of the examples and problems given in this book, the following average values will be used, unless otherwise specified:

For the truss weight—compute from the above formulas.

For the roof covering—12 pounds per square foot of roof surface.

For the snow load—15 pounds per square foot of horizontal area.

The weight of the roof covering, and of the snow which may be upon it, is brought by the purlins to the joints or 'apexes' of the upper chords. The weight of the truss itself is also generally regarded as concentrated at the same points, the larger part of it being actually applied there. At each apex of the upper chord there is hence a load called an 'apex load,' and it may be a 'dead apex load' or a 'snow apex load.'

In the wooden truss whose outlines are shown in Fig. 24, let the span be 48 feet, the rise 10 feet, and the distance between the trusses 12 feet. Then the length of the rafter is $\sqrt{24^2 + 10^2} = 26$ feet. Each main rafter is divided into three equal parts called 'panels.' From the formula the truss weight is found to be 1 670 pounds. The weight of the roof covering on each rafter

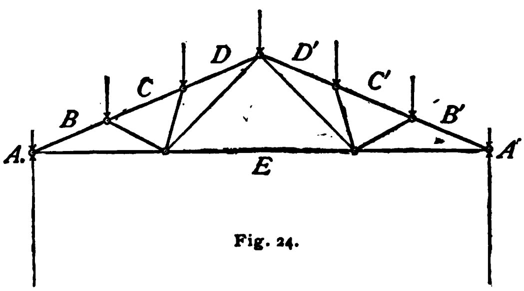


Fig. 24.

is 26 × 12 × 12 = 3 744 pounds. The weight of the snow supported by the entire truss is 48 × 12 × 15 = 8 640 pounds. The total dead load is 1 670 + 2 × 3 744 = 9 158 pounds, and the dead apex loads *BC*, *CD*, *DD'*, *D'C'*, and *C'B'* are each one-sixth of the total load or 1 526 pounds, while *AB* and *B'A'* are each one-half as much, or 763 pounds. The snow apex loads are in like manner 1 440 pounds and 720 pounds. The apex load is also called the 'panel load.'

If the panels be of unequal lengths, the load at any apex is found by considering that the weights brought to it by the purlins are those upon a rectangle extending in each direction half-way to the adjacent apexes.

When the two halves of the truss are symmetrical the reactions of the supports are equal, each being one-half of the total load. When the truss is unsymmetrical, the reactions are found in the same way as for concentrated loads on a beam. In the above example each dead load reaction is 4 578 pounds and each snow load reaction is 4 320 pounds.

The reaction and the half apex load acting at each support produce an effective reaction equal to their difference. This effective reaction is that due to the other apex loads, and therefore the half apex loads at the supports may be omitted entirely from consideration. The effective reaction for the above example is 4 320 − 720 = 3 600 pounds, or by disregarding the loads at the supports it is one-half the sum of the full panel loads, thus, ½(5 × 1 440) = 3 600 pounds.

Prob. 23. A wrought iron truss, like Fig. 24, has a span of 60 feet, rise 14 feet, and distance between trusses 16 feet. Find the apex loads and reactions due to dead and snow loads.

Prob. 24. A wrought iron truss of the above form has a span of 90 feet 6 inches and a rise of 18 feet 9 inches, the trusses being 18 feet apart. Compute the apex loads and reactions.

ART. 17. STRESSES DUE TO DEAD AND SNOW LOADS.

The wooden truss shown in Fig. 25 has a span of 42 feet, rise at peak 14 feet, rise at hip 10 feet 3 inches, horizontal distance from hip to peak 12 feet 6 inches, and a distance between trusses of 8 feet.

The lengths of the members BG and EL are each $\sqrt{8.5^2 + 10.25^2} = 13.32$ feet, and of CH and DK $\sqrt{12.5^2 + 3.75^2} = 13.05$ feet. The weight of the truss is found to be 874 pounds by the formula in Art. 16. The roof covering on BG weighs $13.32 \times 8 \times 12 = 1\,279$ pounds, and on CH is $13.05 \times 8 \times 12 \doteq 1\,253$ pounds. The snow load on BG is $8.5 \times 8 \times 15 = 1\,020$ pounds, and on CH is $12.5 \times 8 \times 15 = 1\,500$ pounds. The apex loads, expressed in pounds, are then as follows:

	$AB = EF$	$BC = DE$	CD	TOTAL
Truss, ...	109	218	218	872
Roof covering, . .	640	1 266	1 253	5 065
Dead, ...	749	1 484	1 471	5 937
Snow, ...	510	1 260	1 500	5 040

Each dead load reaction is therefore 2 969 pounds, and each snow load reaction 2 520 pounds.

The truss diagram, Fig. 25, composed of the center lines of the truss members, is carefully drawn to as large a scale as convenient, each joint being marked by a fine needle point and surrounded by a small circle to limit the lines drawn toward the point. In the actual construction the diagram was drawn to a scale of 3 feet to an inch, but the above figure is reduced to nearly one-seventh of the original size.

The external forces acting upon the truss are in equilibrium, and hence form a closed force polygon (Art. 2). These forces being parallel the resulting polygon becomes a straight line, which is called the 'load line.'

Taking first only the dead load and using a suitable scale (1 000 pounds to an inch was used on the original drawing), the apex loads taken in regular order from left to right are laid off in succession on the vertical load line *af* in Fig. 26, thus: the distance *ab* is laid off equal to the load *AB* or 749 pounds, then *bc* equal to the load *BC*, and so on. Next *fm* is laid off upward equal to the reaction *FM* or 2 969 pounds, and *ma* equal to *MA*, thus closing the polygon.

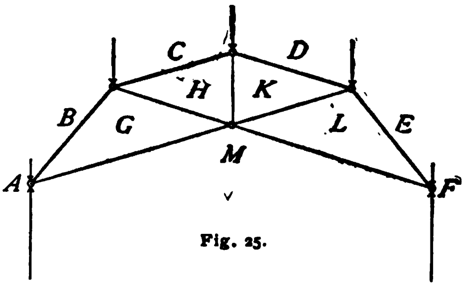


Fig. 25.

Beginning with the joint at the left support where the reaction *MA* and the load *AB* are held in equilibrium by the stresses in *BG* and *MG* the polygon representing these forces will also be closed; *bg* is therefore drawn parallel to *BG* and *mg* parallel to *MG*. The lengths of *bg* and *mg*, measured by the scale used on the load line, give the magnitudes of the stresses in these members. To find the character of the stresses the direction around the polygon indicated by the upward reaction is followed, that is, from *m* to *a*, *a* to *b*, *b* to *g*, and from *g* to *m*. Transferring these directions to the joint considered, the stress in *BG* acts toward the joint and is therefore compression, while that in *GM* acts away from the joint and is tension.

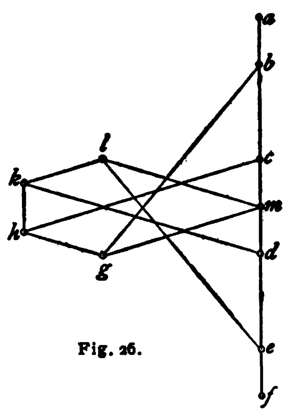


Fig. 26.

The stress diagram is continued by passing to the left hip-joint and constructing the force polygon *bchg* to represent the stresses in the members meeting at that joint. Next are constructed the polygons for the right support, the right hip-joint, and finally for the joint at the peak. The closing line *hk* must be parallel to *HK*, and the entire diagram be symmetrical with respect to a horizontal axis through *m*.

It should be noticed that in following around each polygon the direction is the same as that in which the loads and reactions are taken and laid off on the load line, the direction being in this example that of the hands of a watch. For instance, the direction around the polygon *bchgb* is indicated by the order of the letters, the corresponding capital letters being arranged around the left hip-joint in the same order. Attention to these facts facilitates the construction of the polygons for each joint, and often enables the characters of the stresses to be determined even when none of them have been previously found from another polygon.

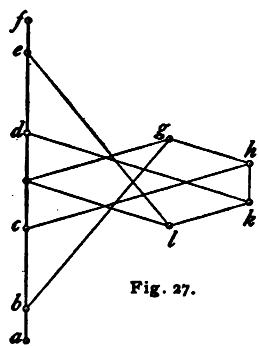


Fig. 27.

Considering now the polygon *chgmabc*, the forces represented by its sides are in equilibrium since the polygon is closed. Referring to Fig. 25 it is seen that the stresses in *CH*, *HG*, and *GM* are in equilibrium with the reaction and loads on the left of a section cutting these members. But *chgmfedc* is also a closed polygon, hence the stresses in the same members are in equilibrium with the reaction and loads on the right of the section. Thus is again demonstrated the principle that the internal stresses in any section hold in equilibrium all the external forces on either side of the section (Art. 7).

The stress diagram for snow load, Fig. 27, is next constructed in a similar manner. As the snow loads are here laid off in the reverse order from the dead loads, the diagram is situated on the right of the load line instead of on the left as in the preceding figure.

The lines in all the diagrams are to be drawn with a well-sharpened pencil pressed lightly on the paper so as to produce a very fine line. As soon as an intersection is obtained it is to be marked with a needle point, enclosed with a small circle, and designated by the proper letter; other lines drawn to or from that point are not to pass within the circumference of this circle. The triangle and straight edge used in drawing parallel lines should be so arranged as to require the triangle to be moved the shortest distance. Special care is needed to hold the pencil at the same inclination from the beginning to the end of a line, or the line will not be strictly parallel to the edge of the triangle.

The following results, expressed in pounds, are now obtained by applying the scale of force to the stress diagrams:

TRUSS MEMBERS.	$BG = EL$	$CH = DK$	$GM = LM$	$GH = KL$	HK
Dead load stresses.	− 3 860	− 3 860	+ 2 580	+ 1 280	+ 750
Snow load stresses.	− 3 500	− 3 640	+ 2 340	+ 1 310	+ 600

When but a slight difference is found between the lengths of any pair of symmetrical lines in the stress diagram the average of the two is to be used; otherwise the diagram should be re-drawn.

As a final check the stresses in CH are computed (Roofs and Bridges, Part I, Art. 5), the lever arm of CH being measured on the truss diagram and found to be 7.27 feet. The stress due to dead load thus obtained is 3 861 pounds, and that due to snow load is 3 640 pounds.

Prob. 25. The upper chord of a wrought iron truss like Fig. 24 is divided into six equal panels, the triangle formed by the middle panel of each main rafter with the adjacent braces being isosceles. The span is 78 feet, the rise 19 feet 6 inches, and the trusses are 16 feet 6 inches apart, center to center. Find the dead and snow load stresses.

ART. 18. A TRIANGULAR ROOF TRUSS.

In the wrought iron truss in Fig. 28 the upper chord is divided into eight equal panels, verticals are drawn through the apexes, and the diagonals slope upward toward the center. Let the span be 80 feet, the rise of peak 15 feet, the lower

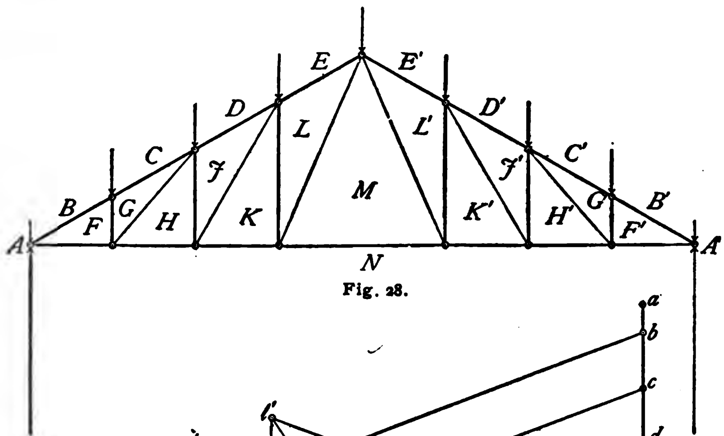


Fig. 28.

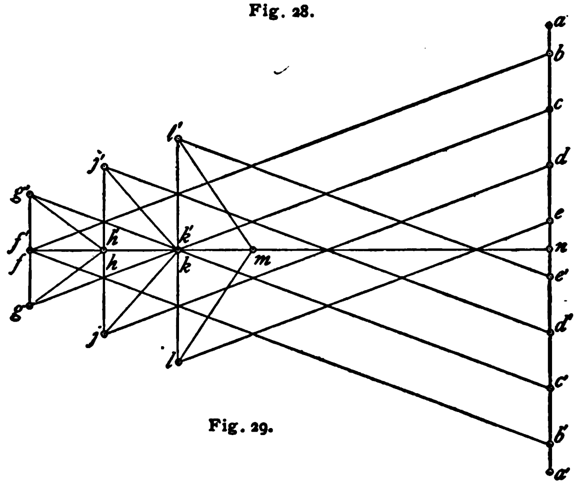


Fig. 29.

chord horizontal, and the distance between adjacent trusses 18 feet, center to center.

The main rafter is found to be 42.72 feet, and each panel 10.68 feet long. The weight of the truss is 9 720 pounds, and of the roof covering 18 455 pounds, making the total dead load

fee.

The
10.68 fee.
of the roof

28 175 pounds. The dead apex load is one-eighth of this amount, or 3 522 pounds, and the half apex loads at the supports are each 1 761 pounds.

Since the inclination of each panel of the upper chord is the same the snow apex loads are equal, each one being $10 \times 18 \times 15 = 2\,700$ pounds. The ratio between any snow apex load and a dead apex load being $\frac{2\,700}{3\,522} = 0.7666$, the stresses caused by the snow load will bear the same ratio to those due to dead load, and hence only the dead load stress diagram is required.

The construction is begun by laying off the load line aa' equal to 28 175 pounds by scale and bisecting it at n. The half apex loads ab and $a'b'$ are next laid off equal to 1 761 pounds and bb' divided into seven equal parts. The reactions are $a'n$ and na, closing the polygon of external forces. The polygons representing the forces acting at each joint are successively formed, as explained in Art. 17, by beginning at the left support, passing to joints alternately on the upper and lower chords until the peak is reached, then going to the right support and passing from joint to joint until the peak is reached again. The last line to be drawn is $l'm$, which must be parallel to $L'M$ and pass through the intersection m of the lines lm and nm. The diagram if accurately drawn will be symmetrical with respect to nf; the distances fh, hk, and km will be equal, the points g, j, and l lying on a line parallel to $f'b'$ drawn through a point below b' on the load line at a distance equal to $b'c'$. Again, cg and $c'g'$ will pass through the intersection k of nf and ll'. As a final check the stress in MN may be computed; thus,

$$(14\,088 - 1\,761)40 - 3\,522(30 + 20 + 10) - S \times 15 = 0;$$

from which the value of S is $+ 18\,784$ pounds.

The scale of this stress diagram should be such that the line bf will not be longer than the main rafter of the truss diagram. The dead load stresses in the following table were obtained by using scales of 5 feet to an inch and 4000 pounds to an inch. The stresses due to snow load are then found by multiplying the dead load stresses by the ratio 0.7666.

Truss Members.		Dead Load Stresses.	Snow Load Stresses.
		Pounds.	Pounds.
Upper chord,	*BF*	− 35 090	− 26 900
	CG	− 35 090	− 26 900
	DJ	− 30 080	− 23 060
	EL	− 25 070	− 19 220
Lower chord,	*FN*	+ 32 870	+ 25 200
	HN	+ 28 180	+ 21 600
	KN	+ 23 480	+ 18 000
	MN	+ 18 780	+ 14 400
Braces,	*FG*	− 3 520	− 2 700
	GH	+ 5 870	+ 4 500
	HJ	− 5 280	− 4 050
	JK	+ 7 060	+ 5 410
	KL	− 7 040	− 5 400
	LM	+ 8 450	+ 6 480

Prob. 26. A wooden truss of the type of Fig. 28 has a span of 60 feet, rise 15 feet, and distance between trusses 14 feet. The upper chord is divided into six equal panels. Find the apex loads, reactions, and the dead and snow load stresses in all the members.

ART. 19. WIND LOADS.

The pressure produced by the wind on a roof surface depends on the direction and velocity of the wind and on the inclination of the roof. The wind is supposed to move horizontally, and a hurricane at 100 miles per hour exerts a pressure of probably 50 pounds per square foot of surface normal to its direction.

In determining the stresses due to wind it is often specified that the wind pressure shall be taken at 40 pounds per square foot of vertical surface.

While the subject is not fully understood, experiments show that the resultant pressure of a horizontal wind on an inclined surface may be represented by a normal force varying with the roof inclination. The following values deduced from experiments give the normal pressure per square foot for a horizontal wind pressure of 40 pounds per square foot for different inclinations of the roof surface:

INCLIN.	NOR. PRESS.	INCLIN.	NOR. PRESS.	INCLIN.	NOR. PRESS.
5°	5.1	25°	22.6	45°	36.0
10°	9.6	30°	26.5	50°	38.1
15°	14.2	35°	30.1	55°	39.4
20°	18.4	40°	33.3	60°	40.0

For all inclinations exceeding 60 degrees the normal pressure is 40 pounds per square foot, and for intermediate inclinations the pressures are obtained by interpolation. Should the horizontal wind pressure be assumed lower or higher than 40 pounds the normal pressure is to be changed in the same ratio.

Let it be required to find the wind apex loads for the truss whose dimensions are given in Fig. 30. The inclination of *AB* is found to be 50° 40′ and that of *BC* 16° 40′, hence from the above table the normal wind pressures are respectively 38.3 and 15.6 pounds per square foot. If the trusses be 12 feet apart the total normal wind pressure on *AB* is

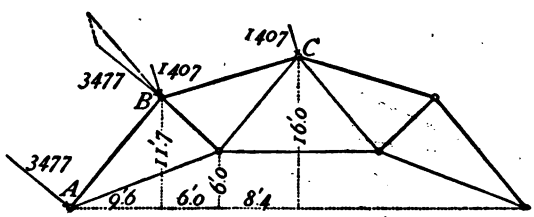


Fig. 30.

$$\sqrt{9.6^2 + 11.7^2} \times 12 \times 38.3 = 6954 \text{ pounds,}$$

one-half of which is applied at A and one-half at B, as shown. In the same way the wind upon BC brings at B and C two normal apex loads, each of 1 407 pounds.

The two apex loads at B are then combined by means of the force triangle, the resultant being 4 711 pounds.

Prob. 27. Find the wind apex loads for the truss in Fig. 25, using the dimensions given in Art. 17.

ART. 20. A TRUSS WITH FIXED ENDS.

Roof trusses of short span, especially wooden trusses, generally have both ends firmly 'fixed' to the supporting walls. The reactions caused by the wind pressure are inclined and their horizontal components tend to overturn the walls of the building.

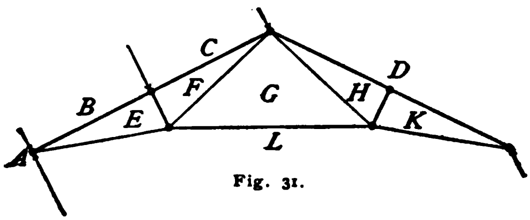


Fig. 31.

Let the truss in Fig. 31 have both ends fixed, the span being 40 feet, the rise of peak 10 feet, the rise of horizontal tie rod 2 feet, and the distance between trusses 12 feet.

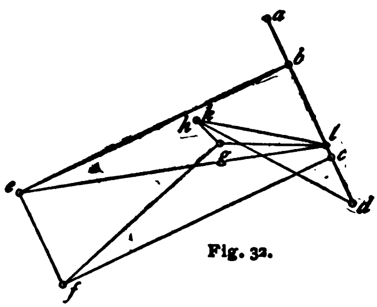


Fig. 32.

The length of the main rafter is found to be 22.36 feet, its inclination 26° 34′ and the normal wind pressure (Art. 19) 23.8 pounds per square foot of roof surface. The apex load BC is

$\frac{1}{2} \times 22.36 \times 12 \times 23.8 =$ 3 193 pounds $=$ 1.6 tons,

and the half apex loads AB and CD are each 0.8 tons.

As the sum of all the external forces in the direction of the wind loads must equal zero, the reactions are parallel to them.

The directions and lines of action of the reactions at the supports being therefore known it is required to find their magnitudes, which is done by the method of Art. 8. The results obtained are 2.2 tons for the left reaction, and 1.0 ton for the right reaction. These values may now be checked by computation. A perpendicular from the right support to the left reaction measures on the truss diagram 35.78 feet. Taking moments about the right support,

$$AL \times 35.78 - 0.8 \times 35.78 - 1.6 \times 24.6 - 0.8 \times 13.42 = 0,$$

whence $AL = 2.2$ tons. In a similar manner, by taking moments about the left support, the reaction DL is found to be 1.0 ton. The same result is obtained both graphically and analytically by replacing the apex wind loads by their resultant, 3.2 tons, applied at the middle of the rafter.

The stress diagram is begun by drawing *ad* normal to the main rafter and equal to 3.2 tons by the scale of force, while *ab* and *cd* are each made one-fourth of the length of *ad*. The diagram is then completed in the manner described in preceding articles. As *h* and *k* are found to coincide it shows that there is no stress in HK when the wind blows on the left side of the truss.

With scales of 3 feet to an inch and 1 ton to an inch the results given in the table were obtained. As a check the stress in GL is computed, the same value being found as that given in the table.

MEMBERS.	STRESSES.
	Tons.
BE	− 4.68
CF	− 4.68
EL	+ 4.88
GL	+ 1.68
EF	− 1.60
FG	+ 3.32
DK	− 2.79
DH	− 2.79
LK	+ 2.08
HK	0
GH	+ 0.52

If the wind loads be placed on the right-hand side of the truss the corresponding stress diagram will be the same as Fig. 32 when revolved about a vertical axis, therefore only one stress diagram is required in this case. Accordingly the stresses in the members of either half of the truss when the wind blows on the right are the same as the stresses in the corresponding members of the other half of the truss for wind on the left side. For instance, the stress in *GH* is + 3.32 and in *EF* is zero for wind on the right.

Prob. 28. Find the wind stresses in all the members of the truss in Prob. 26, both ends being firmly bolted to the walls.

ART. 21. A TRUSS WITH ONE END FREE.

Changes in temperature cause expansion and contraction in iron trusses which if both ends are fixed give rise to certain stresses. To avoid these only one end is fastened to the supporting wall, the otherb eing merely supported or 'free,' so that it may move horizontally in the direction of the length of thetruss.

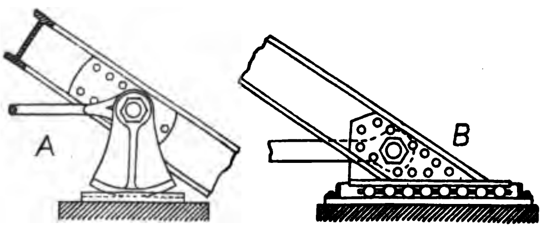


Fig. 33.

The free end may rest upon a smooth iron plate upon which it slides, but this arrangement requires too much friction to be overcome in the case of heavy roofs, especially if the plate becomes rusty. Sometimes it is attached to a rocker, as at *A* in Fig. 33, and often rollers are employed, as shown at *B*. If no friction exists at the free end the reaction there is vertical.

In determining the stresses due to wind when one end is fixed and the other free it is necessary to construct two diagrams, one for the wind blowing on the fixed side and the other for the wind load on the free side.

For example, let the truss in Fig. 34 be taken, the span

being 76 feet, rise of upper chord 19 feet, rise of lower chord 4 feet, and the distance between trusses 15 feet. The web members consist of verticals and diagonals as shown, the chords being divided into eight equal parts.

The inclination of the upper chord is the same as that in the last article, hence the normal wind pressure per square foot

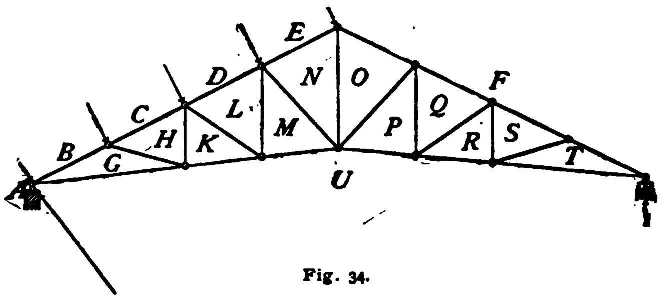


Fig. 34.

is 23.8 pounds, the apex loads *BC*, *CD*, and *DE* are 1.9 tons each, and the loads *AB* and *EF* are 0.95 tons each.

The reaction at the free end being vertical, its value is most readily found by computation. Taking moments about the left support and considering the total wind load concentrated at the middle of the rafter,

$$7.6 \times 21.24 - FU \times 76 = 0, \quad \text{whence} \quad FU = 2.12 \text{ tons.}$$

In Fig. 35 let *af* be drawn perpendicular to the loaded upper chord and made equal to 7.6 tons; *ab* and *ef* are then laid off equal to 0.95 tons each, and *be* divided into three equal parts. The vertical reaction *fu* is next drawn equal to 2.12 tons, and

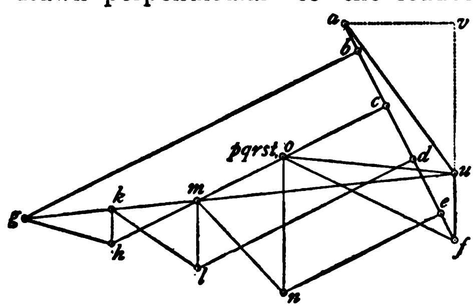


Fig. 35.

as the other reaction must close the force polygon, *ua* represents the magnitude and direction of the reaction *UA* at the fixed end.

The student should notice particularly that it is not possible for the reaction at the fixed end to be parallel to the wind loads when one end rests on rollers.

The stress diagram is constructed in the usual manner by beginning with the forces acting at the left support, and passing to joints alternately on the upper and lower chords. After the joint *DENML* is reached the forces at the peak are taken instead of passing to the central joint of the lower chord. The force polygon for the peak is *nefon*, the point *o* being determined by it. Now passing to the joint below the peak the corresponding force polygon requires a parallel to *UP* to be drawn through *u*, and a parallel to *OP* through *o*. It is found however that the former parallel passes through *o*, thus closing the polygon and reducing *op* to zero. But *uo*, *of*, and *fu* form a closed force triangle which indicates that *O* belongs to the entire space between the chords in the right half of the truss; therefore in this part of the truss each chord has the same stress in every panel and no web member is stressed when the wind blows on the opposite side.

The same thing may be shown in a different manner by beginning at the right support where the reaction *FU* is held in equilibrium by the stresses in *UT* and *FT*. The force triangle *fuo* represents this relation, *uo* being the stress in *UT* and *fo* that in *FT*. Passing to the next joint on the upper chord it is required to draw a triangle two of whose sides shall be parallel to the straight upper chord and the third side parallel to *ST*. This causes the two sides to coincide and the third side to disappear, hence the stress in *FS* equals that in *FT* and the stress in *ST* is zero. The same conditions occur at each joint on the upper and lower chords to the right of the middle of the truss.

If the stress diagram be accurately drawn, the point *m* marks the intersection of *ch*, *ug*, *lm*, and *nm*. The points *g*, *h*, *l*, and *n* will lie in a straight line and be equidistant.

When the wind blows upon the free side of the roof, as in Fig. 36, the apex loads are the same as before, and the reaction

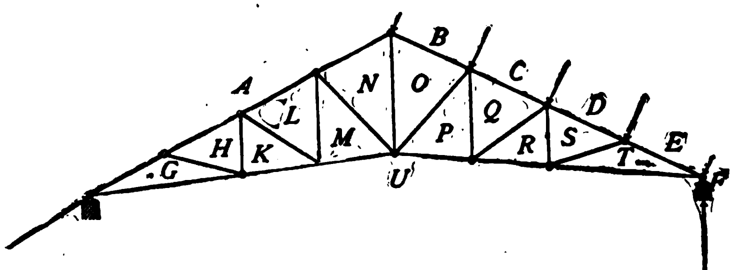


Fig. 36.

FU equals 4.68 tons. After *fa* in Fig. 37 is laid off, *fu* is drawn vertically equal to 4.68 tons by scale, its length being the same as *uv* in Fig. 35. In this stress diagram which is completed similarly to Fig. 35, *an* represents the stress in the upper chord, and *un* that in the lower chord, of the left half of the truss. The braces on the left of *NO* are not affected by the wind on the right.

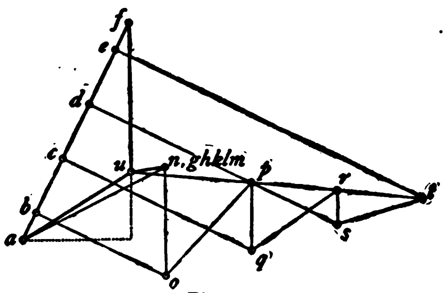


Fig. 37.

The actual construction of the diagrams for this example was made to scales of 6 feet to an inch and 2 tons to an inch, and the results are shown in Figs. 38 and 39. The stresses are marked on the skeleton truss diagrams for convenient comparison, the members in compression being indicated by heavy lines and the tensile members by light lines. The checks by computation give — 6.01 tons for the stress in *FO*, Fig. 34, and — 5.00 tons in *AN*, Fig. 36.

It is seen that greater stresses are produced in the chords except *EN*, and in the center vertical when the wind blows on the

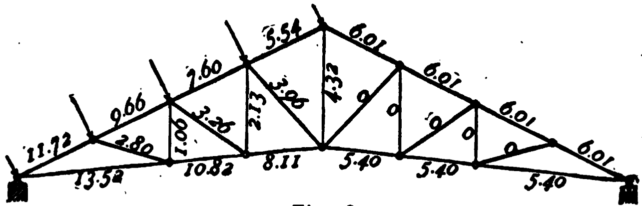


Fig. 38.

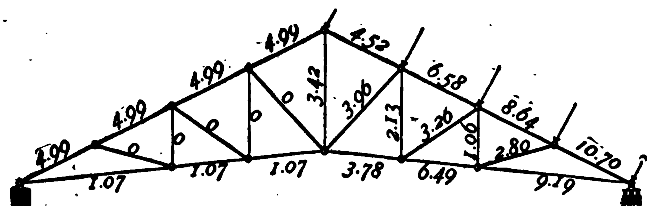


Fig. 39.

fixed side, while the stresses in the braces to the windward with the exception of *NO*, are the same for the wind blowing on either side.

The reactions may also be obtained graphically. The direction of the reaction at the fixed end is unknown, but its line of action passes through the joint at that end, hence the equilibrium polygon should be drawn from that joint to a vertical through the support at the free end. The ray drawn parallel to the closing line will then intersect the vertical through *f* at the point *u*.

Prob. 29. Determine the reactions for the example in this article by the equilibrium polygon.

Prob. 30. Find the wind stresses for the truss in Fig. 28, using the dimensions given in Art. 18. The right end is to rest on rollers.

ART. 22. ABBREVIATED METHODS FOR WIND STRESS.

When the lower chord is horizontal the stresses in the main rafter either to the windward or to the leeward are the same

for the wind blowing either on the fixed or on the free side. The difference between the stresses in the horizontal chord

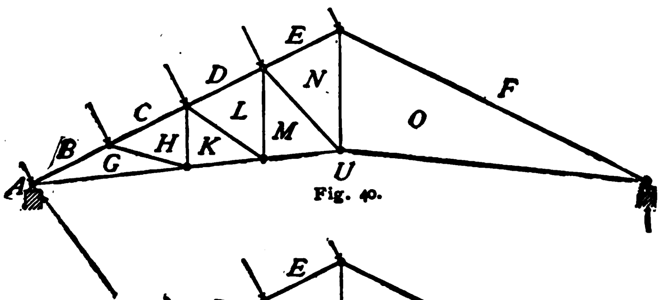


Fig. 40.

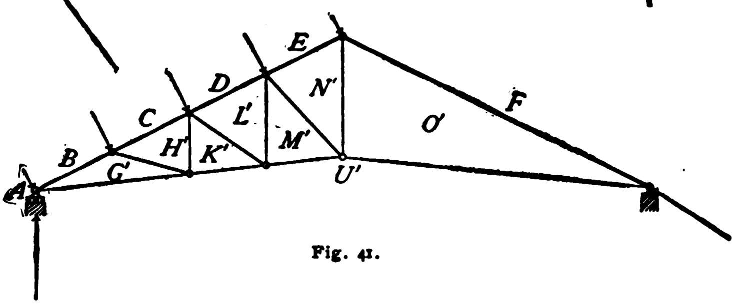


Fig. 41.

under the two conditions equals the horizontal component (*av*, in Fig. 35) of the wind loads, the tension being less when the wind blows on the free side. In such a case, then, only one wind stress diagram is really needed.

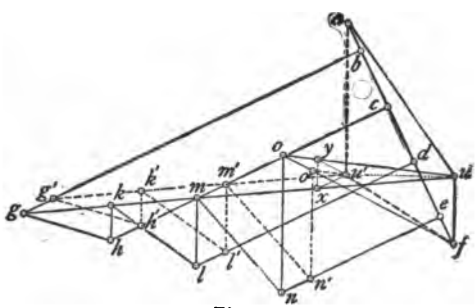


Fig. 42.

Even if the lower chord is raised, a few additional lines on the diagram for wind on the fixed side render the other diagram unnecessary. The full lines in Fig. 42 are simply a copy of Fig. 35 and the truss diagram in Fig. 40 is lettered to correspond, the rollers being at the right support. Now supposing the wind loads to remain unchanged and the rollers to be transferred to the left support, the corresponding stress diagram is that shown in broken lines. This relation of the wind loads, the truss, and the rollers is illustrated in Fig. 41.

Let a horizontal line be drawn through u until its meets the vertical through a at u', and join u' with f and a thus giving the new reactions fu' and $u'a$. Let $u'x$ be drawn parallel to bg meeting ug at x, and $u'y$ parallel to fo meeting uo at y. The line joining x and y will be parallel to no.

The stresses in the main rafter are changed when the rollers are transferred to the left support by the amount $gg' = hh' = ll' = nn' = u'x = u'y = oo'$, those in the lower chord are changed by $ux = uy$, and that in NO by xy, while the remaining stresses are unaltered.

Applying the scale, $u'x$ and $u'y$ are each found to be 1.02 tons, the differences ux and uy are each 4.33 tons and xy measures 0.90 tons. In the following table the first line contains the stresses obtained from Fig. 35, and after subtracting the changes of stress, the same results are obtained as those derived from Fig. 37:

STRESSES FOR WIND ON THE LEFT.

TRUSS MEMBERS, . .	*BG*	*CH*	*DL*	*EN*	*FO*	*UG*	*UK*	*UM*	*UO*	*NO*
Rollers on right, . .	11.72	9.66	7.60	5.54	6.01	13.52	10.82	8.11	5.40	4.32
Changes in stresses, .	1.02	1.02	1.02	1.02	1.02	4.33	4.33	4.33	4.33	0.90
Rollers on left, . . .	10.70	8.64	6.58	4.52	4.99	9.19	6.49	3.78	1.07	3.42
TRUSS MEMBERS, . .	*BG'*	*CH'*	*DL'*	*EN'*	*FO'*	*U'G'*	*U'K'*	*U'M'*	*U'O'*	*N'O'*

For any other type of truss the changes are likewise easily obtained. When the middle panel has a horizontal tie as in the example given on Plate I, the form of the auxiliary polygon $u'xzy$ is somewhat different from the preceding one and the change for the horizontal tie is measured from k to z. The following measurements were obtained from the original diagram which was made with a scale of 2 tons to an inch: $u'x =$

$u'y = 0.88$ tons, $kx = ky = 4.29$ tons, $kz = 4.17$ tons, and $xz = yz = 0.40$ tons.

Prob. 31. Prepare a table of wind stresses similar to the above for the example given on Plate I.

Prob. 32. Find the wind stresses for a wooden truss like Fig. 34 whose span is 74 feet, rise of peak 19 feet, the lower chord being horizontal and the trusses 16 feet 3 inches apart.

ART. 23. COMPLETE STRESSES FOR A TRIANGULAR TRUSS.

On Plate I are given the dimensions of a wrought iron roof truss together with the specified loads. The struts are normal to the rafters as shown on the skeleton outline of the truss. All the diagrams required to determine the stresses due to dead, snow, and wind loads are shown, and are constructed in the manner explained in the preceding articles of this Chapter. The stresses as measured by scale (2 tons to an inch on the original) are arranged in tabular form.

The preliminary computations give the following results:

Length of rafter, ...	41.15 feet.
Weight of truss, ...	3.85 tons.
Weight of roof covering, ...	8.15 tons.
Total dead load, ...	12.00 tons.
Dead apex load, ...	1.50 tons.
Dead load reaction, ...	6.00 tons.
Snow apex load, ...	1.14 tons.
Ratio of snow load stresses to dead load stresses, .	0.76.
Inclination of roof surface, ...	25° 57′.
Normal wind pressure per square foot of roof,	23.3 pounds.
Total wind load, ...	7.92 tons.
Wind apex load, ...	1.98 tons.
Horizontal component of total wind load, . . .	3.47 tons.
Vertical component of total wind load, ...	7.12 tons.
Reaction at free end for wind on fixed side, . .	2.20 tons.
Reaction at free end for wind on free side, . .	4.92 tons.

The reactions are obtained graphically as follows: Supposing both ends of the truss to be fixed, the reactions due to wind on the left side are *au* and *ua* in the stress diagram marked 'wind on fixed side.' They are obtained by the method of Art. 8. The pole is at *o* and the equilibrium polygon is *rst*, the wind apex loads being concentrated at apex 2. But since the right end of the truss rests on rollers the reaction *AK* of the right support is vertical and is represented by the line *ak* or the vertical component of *au*. The closing side *ka* of the force polygon represents the reaction of the left support. Applying the scale the value of *ak* is found to be 2.2 tons, which is also the value of the vertical component of the reaction of the left support when the wind blows on the right side.

In order to design a member for the range of stress (Mechanics of Materials, Art. 81), it is necessary to know the minimum stress as well as the maximum stress to which it is subjected by the combined loads As the dead load always acts its effect must be included in finding both the minimum and the maximum stresses. Snow load always produces stresses of the same kind as the dead load when the rafters are straight, and hence is used only in obtaining the maximum. As the wind cannot blow on more than one side of the roof at the same time, only one of the wind stresses is to be combined with the dead, or with the dead and snow load stresses. If in any member a stress produced by the wind is of a different kind from that due to dead load, the minimum stress equals the algebraic sum of the two, but when, as in the present example, all the stresses in any member are either tensile or compressive, the minimum equals the dead load stress and the maximum equals the sum of the dead, snow, and larger wind load stresses.

It is seen from the table that the maximum chord stresses are greater on the fixed side than on the free side, while the maximum stresses in the bracing are the same on both sides.

Prob. 33. A wrought iron truss of the type shown in Fig. 28 has a span of 76 feet, rise of peak 18 feet, and rise of tie *MN* 3 feet. The trusses are 16 feet 6 inches apart, their right ends resting on rollers. Find the maximum and minimum stresses in all the members.

ART. 24. COMPLETE STRESSES FOR A CRESCENT TRUSS.

In the crescent truss whose outline and general dimensions are given on Plate II the joints on both the upper and lower chords lie on arcs of circles, and the alternate braces are radials of the upper circular arc.

The following dimensions and apex loads are obtained either graphically or by computation, and in some cases by both methods, one being a check on the other:

Radius of circle containing the joints of the upper chord, ...	51.25 feet.
Radius of circle containing the joints of the lower chord, ...	99.06 feet.
Length of panels on the upper chord, ...	11.08 feet.
Weight of truss, ...	4.12 tons.
Weight of roof covering, ...	8.52 tons.
Total dead load, ...	12.64 tons.
Dead apex load, ...	1.58 tons.
Dead load reaction, ...	6.32 tons.
Horizontal projection of panels of upper chord,	8.04, 9.44, 10.53, 10.99 feet.
Snow apex loads, ...	0.48, 1.05, 1.20, 1.29, 1.32 tons.
Snow load reaction, ...	4.68 tons.
Inclinations of roof surface, .	43° 20′, 31° 05′ 18° 30′, 6° 10′.
Normal wind pressures per sq. ft.,	35.2, 27.3, 17.2, 6.2 pounds.
Total wind loads on the panels, . .	3.12, 2.44, 1.52, 0.54 tons.

The dead load stress diagram is constructed in the same way as in previous examples, and is symmetrical with respect to a horizontal axis through *k*. As the snow apex loads are not uniform a separate diagram for stresses due to snow loads is required, and this one is also symmetrical with respect to a horizontal axis through k.

The wind apex loads are next obtained by combining half of the wind loads on the panels adjacent to each apex as illustrated in Art. 19. On the truss diagram in Plate II the wind apex loads are drawn to double the scale of tons in order to determine their directions with greater precision. The reactions are then obtained by means of the equilibrium polygon (Arts. 6, 8, and 21). The ray ou', parallel to the closing side of the polygon, intersects the resultant of the wind loads $au'a$ at the point u' giving au' and $u'a$ as the reactions of the right and left supports if both ends of the truss were fixed, but as the right end is free its reaction ak is vertical and equal to the vertical component of au'. The load line for wind on the free side is obtained from that for wind on the fixed side by revolving it about a vertical axis, which operation may be conveniently performed by means of a piece of tracing paper. The point u'' in this diagram is the same as u' in the preceding one, hence only one equilibrium polygon is required. The reaction ka of the right support is the vertical component of $u''a$.

It is observed that when the wind blows on the free side of the truss, it causes compression in the lower chord from KH' to the left end. The stresses in the upper chord are also considerably less than for wind on the fixed side, and if the rise were a little greater it would cause tension in one or more panels near the left support.

The truss diagram for this example should be twice as large as that on Plate II so that no line in any stress diagram would be longer than the truss member to which it is parallel. This relation was here disregarded as the limited size of the plate would have reduced the stress diagrams to indistinctness.

Unless special care is exercised in drawing the wind stress

diagrams they will not close. As they lack the check of symmetry it is not so easy to locate the error, and therefore it is best to construct new diagrams until one is obtained that closes properly. In all cases the work should proceed from each support toward the center of the truss.

Upon the completion of the stress diagrams their lines are measured by the scale of force, the stresses arranged in a table and the maximum and minimum stresses found, as explained in the preceding article. For instance, the maximum stress in *KD* is $+ 10.8 + 8.5 + 9.7 = + 29.0$ tons, and its minimum stress is $+ 10.8 - 3.6 = + 7.2$ tons. The maximum stress in *GH* is $- 0.1 + 2.3 = + 2.2$ tons and the minimum stress in the same member is $- 0.1 - 0.1 - 1.7 = - 1.9$ tons.

Prob. 34. Find the maximum and minimum stresses in all the members of Fig. 30, the right end resting on rollers.

ART. 25. AMBIGUOUS CASES.

When, in the determination of the stresses in the Fink truss shown in Fig. 43, the joint at the middle of the rafter is reached the load *BC* and the stresses in *BG* and *GH* are known, leaving three stresses unknown, namely, those in *CL*, *LK*, and *KH*. As the resultant of the load and the known stresses cannot be resolved into more than two given directions, another condition needs to be added.

If the loads *AB* and *CD* be equal, as in this example, the symmetrical relation of *GH* and *LK* causes them to have equal stresses and therefore *fg* and *lm* lie in the same straight line parallel to *hk*. The polygon *hgbclkh* is then readily completed. If the loads *AB* and *CD* be unequal, the panels remaining equal on the upper chord, the polygon may be drawn by noting that the point *k* must lie midway between the parallels *cl* and *dm*. This follows from the fact that *LM* is normal to the upper chord and that *KL* and *MN* are of equal length and make the same angle with *LM* as well as with the upper chord. The triangle *lkm* is hence an isosceles triangle.

If *CL* and *DM* be of unequal lengths then both of these methods fail. A general solution of this problem was given by

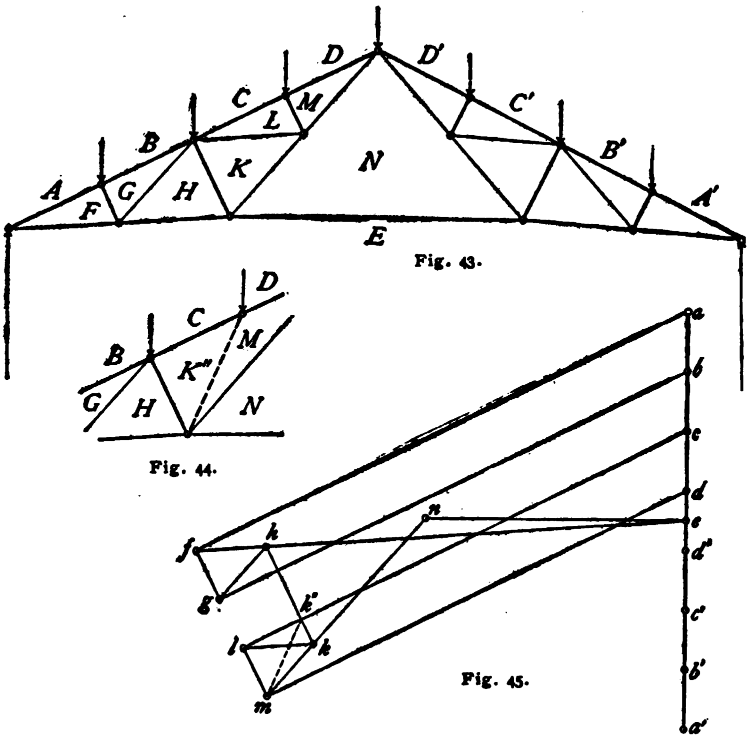


Fig. 43.

Fig. 44.

Fig. 45.

WILLETT in a paper read before the Chicago Chapter of the American Institute of Architects, March 22, 1888, which consists in temporarily changing the webbing of the truss.

Let the braces *KL* and *LM* be removed and the diagonal *K''M* be substituted as shown in Fig. 44. The load *BG* and the stresses in *BG* and *GH* being known, those in *HK''* and *K''C* are found from the polygon *hgbck''h* in Fig. 45. For the next apex on the upper chord the polygon is *k''cdmk''*, the

line mk'' being the unknown stress thus determined. Passing to the joint $HK''MNE$ where three stresses are now known, the polygon $ehk''mne$ gives the unknown stresses mn and ne. As the line mn is now fixed, the original webbing is restored and the remaining parts of the stress diagram drawn ; hk'' is produced to meet mn at k, and kl and ml are drawn parallel to KL and ML respectively to meet ck'' produced at l.

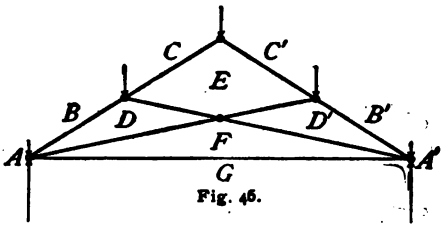


Fig. 46.

The remaining half of the diagram is not shown in Fig. 45. If each half of the truss and the loading upon it be equal to the other, the complete stress diagram will be symmetrical with respect to a horizontal axis through e.

In the truss whose outline is given in Fig. 46 it is not possible to begin the stress diagram by considering the forces acting at the left support, since three unknown stresses hold in equilibrium the known reaction and half apex load.

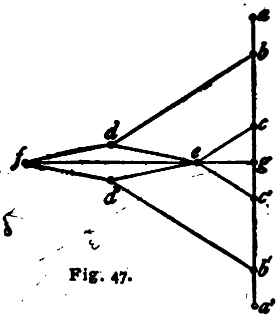


Fig. 47.

At the peak the load CC' is supported by two members whose stresses are $c'e$ and ec in Fig. 47. The quadrilateral $bced$ gives the stresses in the members meeting at the joint $BCED$, and $c'b'd'e$ gives those for the corresponding joint in the right half of the truss. Passing to the joint below the peak the stress polygon is found to be $ded'f$. For the left support the reaction, half apex load, and the stresses bd and df are known, but one stress remains unknown. As equilibrium exists at this joint the polygon must be closed by the line fg, which is also to be parallel to FG. This completes the diagram, and by following around the polygons all the stresses are found to be compression except *fg*, which is tension.

Suppose the tie *FG* to be omitted. Each reaction must then be inclined in order to maintain equilibrium, its horizontal component being equal to *fg*. The reaction of the right support is therefore *a'f* (not drawn) and that of the left support *fa*.

Prob. 35. Find the dead load stresses in the wooden truss of Fig. 48, the span being 36 feet, the rise 12 feet, and the distance between trusses 12 feet. Then find the stresses for a truss like Fig. 46 of the same dimensions and compare the results.

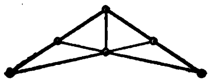


Fig. 48.

Prob. 36. A wooden truss like Fig. 49 has a span of 48 feet and a rise of 12 feet, both ends being fixed. Find the maximum and minimum stresses, the trusses being 14 feet apart.

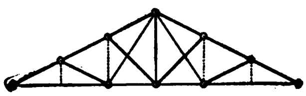


Fig. 49.

ART. 26. UNSYMMETRICAL LOADS AND TRUSSES.

In mills and shops, loads are frequently suspended from the lower chords of the roof trusses, as for instance, lines of shafting, etc. In such cases it is convenient to determine the stresses due to the loads by means of a separate diagram, as illustrated in Fig. 50. The load *AB* produces no stress in *DE* or *FG*, while it causes stresses in the other members of the same kind as those due to dead, snow, and wind loads, and hence their maximum stresses are increased. In special cases such sus-

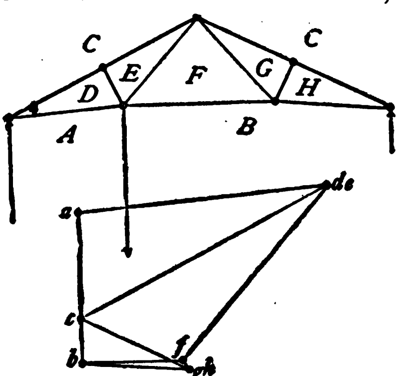


Fig. 50.

pended loads may even change some maximum stresses due to the other loads from compression to tension or from tension to compression.

When a ceiling is attached to the lower chord it becomes a part of the dead load and needs no separate diagram. It is combined at once with the other loads in the dead load diagram, all the loads and reactions being taken in regular order around the truss and laid off on the load line, some portions of which will be found to overlap.

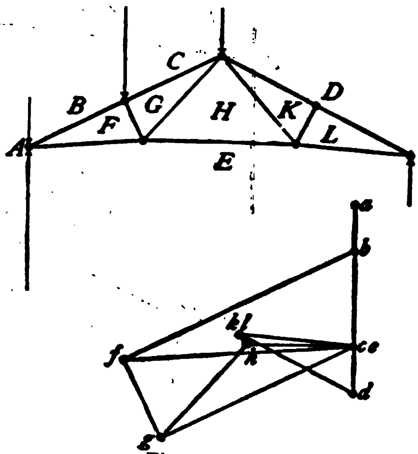


Fig. 51.

It was formerly the practice in England to find the effect of the wind on roof trusses by taking vertical loads of 20 pounds or more per square foot of horizontal projection, acting upon one side of the roof. Fig. 51 shows a truss under such loads and the resulting stress diagram. It is observed that the stress thus caused in *BF* is greater than that in *CG*, while under normal wind loads they are equal.

In Fig. 52 is given an unsymmetrical truss under the action of dead load. The stress diagram is hence unsymmetrical and has also fewer checks upon its construction. The main check however still remains, which requires that after working from each support toward the peak the closing line *op* shall be parallel to the member *OP*. To determine the stresses in an unsymmetrical truss by the analytic method materially increases the labor of computation required for a symmetrical truss, but with the graphic method it makes no difference whatever for any type of truss.

Prob. 37. Let the load *AB* in Fig. 50 be 1.75 tons, and the dimensions of the truss the same as given in Art. 20. Find the stresses in all the members.

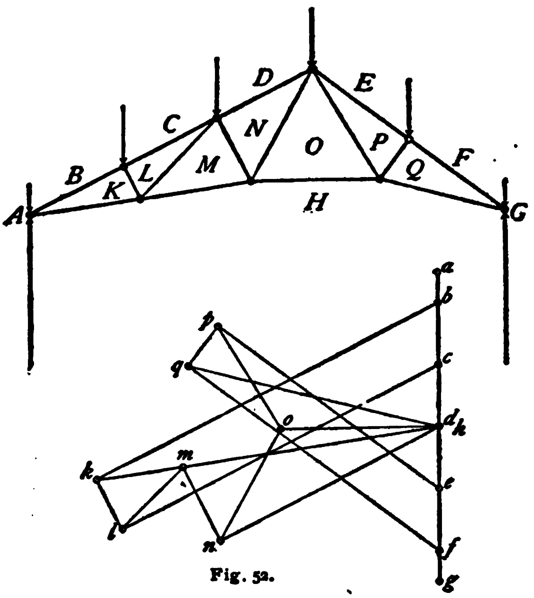


Fig. 52.

Prob. 38. Find the wind stresses for the same truss under a vertical wind pressure of 20 pounds per horizontal square foot, and compare the results with those obtained in Art. 20.

CHAPTER III.

BRIDGE TRUSSES.

ART. 27. LOADS ON BRIDGE TRUSSES.

The weight of the floor, lateral bracing, trusses, and all the pieces that unite and stiffen them, compose the dead load of a bridge. This weight depends upon the span, width, and style of the bridge, and upon the live load and unit stresses adopted in its design, and varies considerably in individual cases; it is usually lighter for a highway bridge than for a railroad bridge.

The total weight or dead load of a highway bridge with two trusses may be expressed approximately by the following empirical formula:

$$w = 140 + 12b + 0.2bl - 0.4l,$$

in which w is the weight in pounds per linear foot, b the width of the bridge in feet (including sidewalks, if any), and l the span in feet. The width of a highway bridge in the clear varies from 16 to 24 feet, which is only exceeded in large cities.

The total dead load of a railroad bridge for a standard gauge track may be approximately found from the following empirical formulas:

$$\text{For single track, } w = 560 + 5.6l,$$
$$\text{For double track, } w = 1\,070 + 10.7l.$$

For spans not exceeding 300 feet these formulas give values usually a little larger than the actual weights, but sufficiently exact for the determination of the stresses. For spans greater than 300 feet they should not be used. The width in the clear between the trusses of a through railroad bridge is about 13 or

14 feet for a single track, and 25 feet for a double track. Wooden and iron bridges of the same strength do not materially differ in weight.

The live load is that which passes over the bridge, and consists of wagons and foot passengers on highway bridges and trains on railroad bridges.

The live loads usually assumed for highway bridges, in pounds per square foot of floor surface, are as follows:

	For Country Bridges.	For City Bridges.
Spans under 50 feet,	90	100
Spans 50 to 125 feet,	80	90
Spans 125 to 200 feet,	70	80
Spans over 200 feet,	60	70

This maximum load consists of a dense crowd of people covering the roadway and sidewalks, and as there is less liability to crowds on long spans as compared with short spans, and for country bridges as compared with those in the city, the load is varied accordingly. Each truss supports one-half of the whole load.

By multiplying the given weight per square foot by the clear width of roadway and sidewalks the live load per linear foot of bridge is obtained. This load is to be placed so as to produce the largest possible stress in any truss member considered.

The live load for a railroad bridge is that of the heaviest cars and locomotives which pass, or are to pass, over it. When a bridge is to be designed these loads are generally specified by the railroad company. Several methods of stating the live load are in use:

1st. A uniform load per linear foot of single track, the shortest spans having the heaviest loads, and about as follows:

Span,	50,	100,	150,	200,	300,	400	feet.
Load,	4 200,	3 600,	3 200,	3 000,	2 600,	2 400	pounds.

This method was formerly much used, but is now only occasionally employed for computing the chord stresses.

2d. A uniform train load, varying as above, which is preceded by one panel of heavy locomotive load. If p be the length of the panel in feet this preceding load in pounds is for a single track often taken as $30\,000 + 3\,500p$. Sometimes the locomotive load is used for two or three panels in front of the train instead of for one.

3d. A uniform train load, varying as above, preceded by one or two locomotives with their tenders, the weight of these being taken as concentrated upon the drivers and other wheels. This style of loading will be fully explained in Art. 39.

If the bridge have only one track each truss sustains but one-half the loads above given, but if it have two tracks each truss sustains the loads as stated, for it might happen that both tracks would be covered at the same time.

Prob. 39. A highway bridge in the country has a span of 88 feet, a roadway 16 feet wide, two sidewalks each 4 feet wide, and each truss has 9 panels. Find the dead and live panel loads.

ART. 28. DEAD LOAD STRESSES.

The principle of the force polygon used in the last chapter for the determination of stresses in roof trusses may often be advantageously employed for the analysis of bridge trusses.

As an illustration let a through Pratt truss for a highway bridge be taken, having 8 panels, a span of 176 feet, and a depth of 26 feet. Let the bridge have a roadway 21 feet wide and two sidewalks each 6 feet wide. By the formula in Art. 27 the dead load per linear foot of bridge is found to be 1 627 pounds, and the dead panel load per truss is 8.95 short tons. It is required to determine the stresses due to this dead load, all being supposed to be on the lower chord.

In Fig. 54 let *qy* be laid off by scale equal to $8.95 \times 7 = 62.65$ tons, let it be divided carefully into 7 equal parts, lettered in

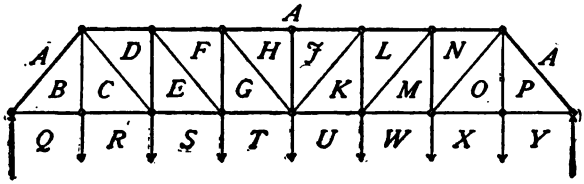


Fig. 53.

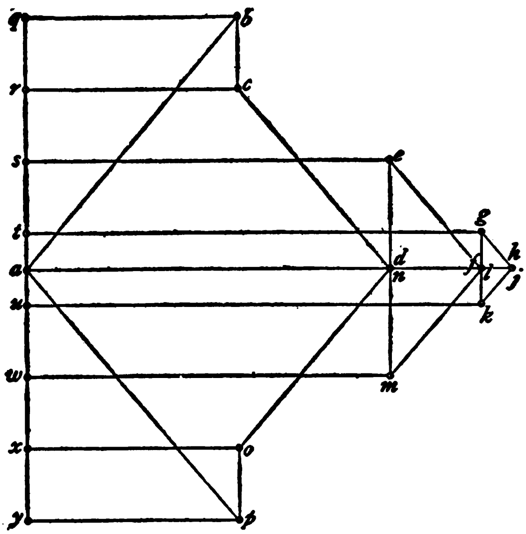


Fig. 54.

the manner indicated and bisected at *a*. The effective reactions are *ya* and *aq*, the half panel loads at the supports being omitted for convenience. At the left support the reaction *AQ* is held in equilibrium by the stresses in *QB* and *BA*, and by Art. 1 these will form the closed force triangle *aqb*. As *aq* acts in the direction from *a* to *q* the other forces must act in the same direction around the triangle, that is, from *q* to *b* and from *b* to *a*. Transferring these directions to the joint *AQB* the stress in *QB* acts away from the joint and is therefore tension, while the stress in *BA* acts toward the joint and is compression. The construction of the stress diagram is continued by passing to the joints alternately on the lower and upper chords until the middle of the truss is reached, then beginning at the right support and passing to the joints in the opposite direction until the diagram closes. If

accurately drawn the diagram will be symmetrical with respect to *ah*. The polygon *qrcb* being a rectangle shows the tension in *CB* to be equal to the load *QR*, and the tension in *RC* to be equal to that in *BQ*. The rectangle *esad* shows the stresses in *AD* and *ES* to be equal in magnitude, the former being compression and the latter tension as previously determined. Again, *af* equals *gt* for a similar reason. It is seen therefore that the stresses in any two chord members whose adjacent spaces are separated only by a vertical have the same magnitude.

The compression in the upper chord increases toward the middle of the truss, and the same is true of the tension in the lower chord. The diagonals are all in tension but *AB*, while the verticals are all in compression but *BC*. In the web members the stresses increase from the middle toward the ends of the truss, with the exception of *BC*, which only serves to transfer the load *QR* to the upper chord.

The following results were obtained from a stress diagram drawn to a scale of 8 tons to an inch :

TRUSS MEMBERS.		STRESSES.	TRUSS MEMBERS.		STRESSES.
		Tons.			Tons.
Upper chord	*AD*	− 45.4	Diagonals	*AB*	− 41.1
	AF	− 56 8		*CD*	+ 29.3
	AH	− 60.6		*EF*	+ 17.6
				GH	+ 5.9
Lower chord	*BQ*	+ 26.5	Verticals	*BC*	+ 9.0
	CR	+ 26.5		*DE*	− 13.5
	ES	+ 45.4		*FG*	− 4.5
	GT	+ 56.8		*HJ*	0

As a final check the stress in *AH* is computed thus:

$$31.325 \times 88 - 8.95\,(66 + 44 + 22) + AH \times 26 = 0,$$

whence $AH = -60.6$ tons.

If instead of concentrating all the dead load on the lower chord, it be divided so that panel loads of 2.95 tons be applied on the upper chord and 6.0 tons on the lower, the stress diagram assumes the form shown in Fig. 56, the broken lines referring to stresses in the right half of the truss. The loads and reactions are taken in regular order around the truss and laid off in succession on the load line. The lower panel loads are taken from left to right, then the right reaction followed by the panel loads on the upper chord from right to left, and finally the left reaction which closes the polygon. This polygon is *efghh'g'f'e'a'b'c'd'dcbae.* The stresses obtained are marked on the right half of the truss diagram, compression being indicated by the heavy lines and tension by the light lines.

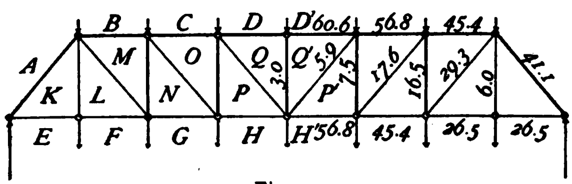


Fig. 55.

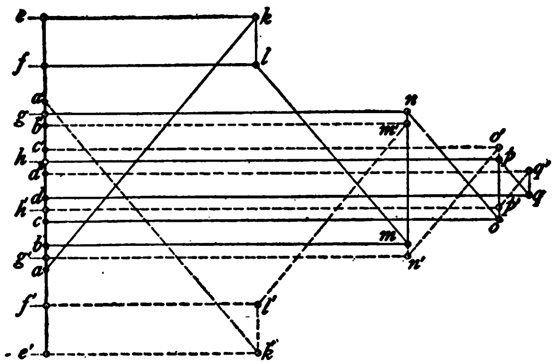


Fig. 56.

Comparing Fig. 56 with Fig. 54 it is observed that all the stresses are the same except those in the verticals whose compression is increased by 2.95 tons—the weight of the upper panel loads. The tension in *KL* is accordingly diminished by the same amount.

A further examination of Fig. 56 shows that the vertical component of *ak* is *ae*, the reaction; the vertical component of *lm* is $ae - ef - ba$; $mn = ae - ef - fg - ba$; and so on. Therefore the following principle is established :

For trusses with horizontal chords the vertical component of the stress in any web member equals the reaction minus all the loads on the left, that is, equals the vertical shear for that member.

The only exception to this is the vertical *KL* for reasons already given in the first part of this article. The above principle may be derived from the relation existing between the stresses in any section of a truss and the external forces on either side of that section as demonstrated in Arts. 7 and 17.

The diagram also shows that the difference between the magnitudes of the stresses in any two chord members equals the sum of the horizontal components of the stresses in the web members situated between them. For instance, the difference between *hp* and *gn* is the horizontal component of *no*, which also equals the difference between *co* and *gn* or between *ph* and *bm*. The horizontal component of any diagonal is called a chord increment and forms the base of a right triangle whose height is the vertical shear in that diagonal. (Roofs and Bridges, Part I, Art. 26.)

Prob. 40. A through Pratt truss of a single track railroad bridge consists of 7 panels, each 23 feet 2 inches long and 25 feet deep. One-third of the dead load being on the upper chord, find the stresses in all the members.

ART. 29. LIVE LOAD STRESSES IN A WARREN TRUSS.

As every load placed upon a bridge truss produces compression in the upper chord and tension in the lower chord, the greatest chord stresses produced by a live load occur when every panel point of the chord supporting the floor beams is loaded. The chord stresses due to a uniform live load are hence obtained from a diagram exactly similar to that for a dead load applied only upon one chord. Hence the stress in any chord member, due to a uniform live load, bears the same ratio to the dead load stress as that of the corresponding apex loads, and accordingly either stress may be derived from the other by using this constant ratio.

In order to investigate the effect of live load on the web members, let a deck Warren truss of 7 panels be taken, the span being 126 feet, the depth 12 feet, and the live load 1 700 pounds per linear foot per truss. The live panel load is then 15.3 tons.

Placing a panel load at apex 1 in Fig. 57, the stresses due to this single load are obtained by drawing Fig. 58 in the usual

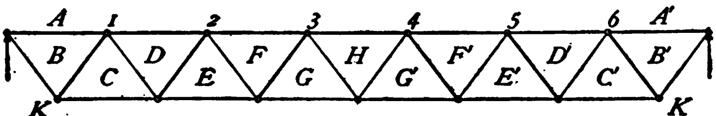


Fig. 57.

manner. The reaction $a'k$ is one-seventh of the panel load. The stresses in the braces are found to be alternately compression and tension each way from the load, and on either side the stresses are the same in magnitude from the load to the support, their vertical components being equal to the reaction on that side.

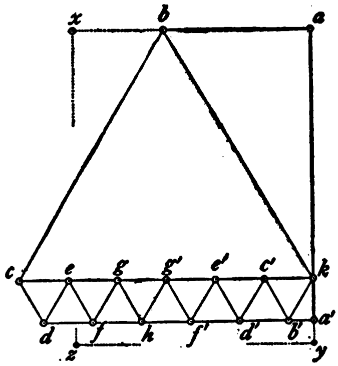


Fig. 58.

For a panel load at apex 2 the reaction of the right support will be twice as great as for the load at 1, and hence the stresses in all the braces on the right of apex 2 will also be twice as large; for a load at apex 3 the stresses on its right will be three times as great as for the load at 1, and so on. Again, a panel load at apex 6 will produce the same stresses on its left as the load at 1 caused on its right, and a load at 3 will produce stresses in the braces on its left equal to four times those due to the load at

6. The stress in each web member due to a single live panel load at any apex may therefore be obtained by taking a simple multiple of the stress for that member as given by Fig. 58.

In the following table the first and sixth lines are thus filled out directly with the results scaled off from the diagram (which was originally drawn to a scale of 5 tons to an inch), and the other lines by taking multiples of these as indicated above. The stresses in each column are then combined so as to give the greatest and least stresses and those due to a uniform live load throughout.

WEB MEMBERS.	*K'B*	*BC*	*CD*	*DE*	*EF*	*FG*	*GH*
Live panel load at 1,	+16.38	−16.38	−2.73	+2.73	−2.73	+2.73	−2.73
2,	+13.65	−13.65	+13.65	−13.65	−5.46	+5.46	−5.46
3,	+10.92	−10.92	+10.92	−10.92	+10.92	−10.92	−8.19
4,	+8.19	−8.19	+8.19	−8.19	+8.19	−8.19	+8.19
5,	+5.46	−5.46	+5.46	−5.46	+5.46	−5.46	+5.46
6,	+2.73	−2.73	+2.73	−2.73	+2.73	−2.73	+2.73
Live load, greatest,	+57.33	−57.33	+40.95	−40.95	+27.30	−27.30	+16.35
Live load, least,	0	0	−2.73	+2.73	−8.19	+8.19	−16.35
Uniform live load,	+57.33	−57.33	+38.22	−38.22	+19.11	−19.11	0

It is found that for any given diagonal all the loads on one side of it cause one kind of stress, while those on the other side cause the opposite stress. The maximum stress is hence produced in a web member when the live load covers the larger segment of the span, and the minimum stress when the smaller segment is loaded.

In the construction of stress diagrams for a truss with horizontal chords and equal panels it is not necessary to draw the skeleton outline of the truss to a large scale. If in this example ax be laid off by the linear scale equal to some convenient multiple of the half panel length and ay equal to the same multiple of the depth of truss, xy will give the direction of half the web members, and in transferring this direction the triangle will require very little shifting along a straight edge, thus promoting accuracy. The line *ay* should be longer than *ak*. Completing the rectangle *xayz*, the direction of the remaining braces will be given by *az*.

The results in the line 'uniform live load' in the table should be the same as those derived from a stress diagram made for a live panel load at every panel point or apex, and may thus be checked. As such a diagram is required for the chord stresses it will also answer this purpose.

As the live load cannot act alone, but always in conjunction with the dead load, the stresses due to the combined loads are required. These are given in the following table. The dead apex load on the lower chord is 1.71 tons, and on the upper chord 3.42 tons. The dead load stresses were obtained from a diagram of 5 tons to an inch.

WEB MEMBERS.	*KB*	*BC*	*CD*	*DE*	*EF*	*FG*	*GH*
Live load, greatest,	+ 57.33	− 57.33	+ 40.95	− 40.95	+ 27.30	− 27.30	+ 16.35
Live load, least,	0	0	− 2.73	+ 2.73	− 8.19	+ 8.19	−16.35
Dead load,	+ 20.35	− 18.18	+ 13.85	− 11.68	+ 7.44	− 5.34	+ 1.07
Maximum,	+ 77.68	− 75.51	+ 54.80	− 52.63	+ 34.74	− 32.64	+ 17.42
Minimum,	+ 20.35	− 18.18	+ 11.12	− 8.95	− 0.75	+ 2.85	− 15.28

The stresses due to the combined load are obtained by adding the dead load stresses to each of the corresponding live load stresses.

By comparing these results with the computations for the same example in Roofs and Bridges, Part I, Art. 34, it is seen that they are correct to or within one-tenth of a ton, which is sufficiently accurate for all purposes of design.

The same method of tabulation might be applied to the

chord stresses, but the diagram for a full live load can be made in less time.

Prob. 41. Find the maximum and minimum chord stresses for the above example.

ART. 30. LIVE LOAD STRESSES IN A PRATT TRUSS.

If a Pratt truss were built having only those diagonals which are strained under dead load it would be necessary that some of them resist the compression produced by certain positions of the live load. As, however, the diagonals are only to be subjected to tension this is prevented by inserting other diagonals inclined in the opposite direction. Panels having two diagonals are said to be counter-braced and the additional diagonals are called counter-ties or counter-braces. The main and counter brace in any panel cannot both be strained at the same time by any system of loading.

When the counter-ties are called into action by the live load the stresses in the adjacent verticals are different from what they would be provided the main braces could withstand compression. This can readily be seen by changing any diagonal and making the corresponding alteration in the stress diagram.

Let the Pratt truss whose dead load stresses were determined in Art. 28 be again considered. It consists of 8 panels each 22 feet long and 26 feet deep. The total width of the bridge, including sidewalks, is 33 feet. Taking the live load at 80 pounds per square foot of floor surface the panel load per truss is

$$\tfrac{1}{2} \times 22 \times 33 \times 80 = 29\,040 \text{ pounds} = 14.52 \text{ tons.}$$

The truss diagram, Fig. 59, is drawn with the diagonals all inclined one way, the main ones being on the left of the center and the counters on the right. Placing one live panel load *RR* at apex 1 the stress diagram, Fig. 60, is constructed which gives all the stresses due to this load. Fig. 61 gives the stresses due to a live panel load at apex 7. After *rr* is laid off equal to the panel load of 14.52 tons, *ru* is marked off, by a suitable linear scale, equal to 26 feet and *rt* equal to 22 feet, then *ut* gives the inclination of the diagonals. Drawing *rs* parallel to *ut* it is found

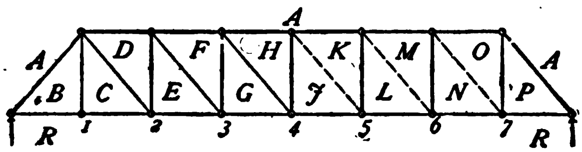


Fig. 59.

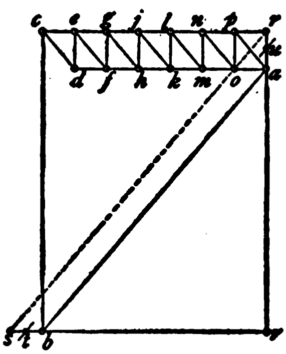


Fig. 60.

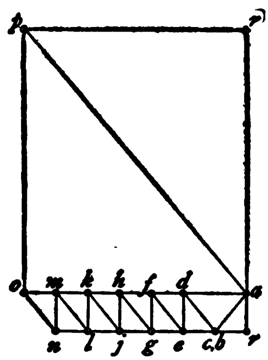


Fig. 61.

to measure 19.02 tons. In both Fig. 60 and Fig. 61 the smaller vertical lines are one-eighth of the length of *rr*, or 1.82 tons, and the smaller diagonals are one-eighth as long as *rs*, or 2.38 tons. The only part of the stress diagrams actually required consists of the similar right triangles *urt* and *rrs*, and the line *rs* is to be carefully determined. Unless these triangles are very nearly of the same size, as in the above example, the latter should be made larger than the former.

The live load stresses are then tabulated as explained in Art. 29. The dead load stresses are obtained from Fig. 62, one-half of which is like the same part of Fig. 56, while the other half is changed so as to give the stresses when the counter-

braces alone are inserted in the right half of the truss as shown in Fig. 59. Two lines in this diagram are marked *hj*, the upper one measures 3.0 tons and represents the compression in *HJ* when the main ties act on each side of it (which occurs under a full live load), and the lower line measures 1.5 tons being the tension in *HJ* when the main tie acts on the left and the counter-tie on the right, as indicated in the truss diagram.

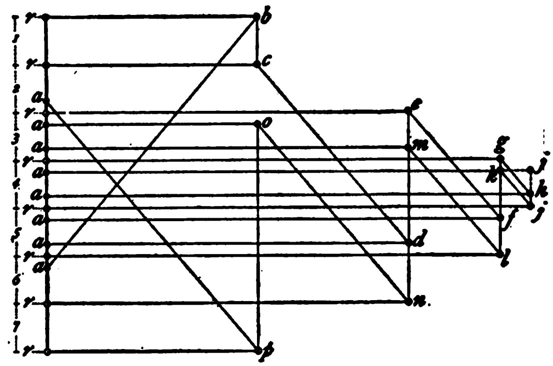


Fig. 62.

All of the tabulated results except those in the last two lines of each table were obtained as if the web members in Fig. 59

Truss Members.	End Posts.	Main Ties.			Counter-Ties.		
	AB = *PA*	*CD*	*EF*	*GH*	*JK*	*LM*	*NO*
Live panel load at 1	− 16.7	− 2.4	− 2.4	− 2.4	− 2.4	− 2.4	− 2.4
2	− 14.3	+ 14.3	− 4.8	− 4.8	− 4.8	− 4.8	− 4.8
3	− 11.9	+ 11.9	+ 11.9	− 7.1	− 7.1	− 7.1	− 7.1
4	− 9.5	+ 9.5	+ 9.5	+ 9.5	− 9.5	− 9.5	− 9.5
5	− 7.1	+ 7.1	+ 7.1	+ 7.1	+ 7.1	− 11.9	− 11.9
6	− 4.8	+ 4.8	+ 4.8	+ 4.8	+ 4.8	+ 4.8	− 14.3
7	− 2.4	+ 2.4	+ 2.4	+ 2.4	+ 2.4	+ 2.4	+ 2.4
Uniform live load,	− 66.7	+ 47.6	+ 28.5	+ 9.5	− 9.5	− 28.5	− 47.6
+ Total,	0	+ 50.0	+ 35.7	+ 23.8	+ 14.3	+ 7.2	+ 2.4
− Total,	− 66.7	− 2.4	− 7.2	− 14.3	− 23.8	− 35.7	− 50.0
Dead load,	− 41.1	+ 29.3	+ 17.6	+ 5.9	− 5.9	− 17.6	− 29.3
Maximum,	− 107.8	+ 79.3	+ 53.3	+ 29.7	+ 8.4	0	0
Minimum,	− 41.1	+ 26.9	+ 10.4	0	0	0	0

Truss Members.		Verticals.						
		BC	*DE*	*FG*	*HJ*	*KL*	*MN*	*OP*
Live panel load at	1	+ 14.5	+ 1.8	+ 1.8	+ 1.8	+ 1.8	+ 1.8	+ 1.8
	2	0	+ 3.6	+ 3.6	+ 3.6	+ 3.6	+ 3.6	+ 3.6
	3	0	− 9.1	+ 5.5	+ 5.5	+ 5.5	+ 5.5	+ 5.5
	4	0	− 7.3	− 7.3	+ 7.3	+ 7.3	+ 7.3	+ 7.3
	5	0	− 5.5	− 5.5	− 5.5	+ 9.1	+ 9.1	+ 9.1
	6	0	− 3.6	− 3.6	− 3.6	− 3.6	+ 10.9	+ 10.9
	7	0	− 1.8	− 1.8	− 1.8	− 1.8	− 1.8	+ 12.7
Uniform live load,		+ 14.5	− 21.9	− 7.3	+ 7.3	+ 21.9	+ 36.4	+ 50.9
+ Total,		+ 14.5	+ 5.4	+ 10.9	+ 18.2	+ 27.3	+ 38.2	+ 50.9
− Total,		0	− 27.3	− 18.2	− 10.9	− 5.4	− 1.8	0
Dead load,		+ 6.0	− 16.5	− 7.5	[− 3.0] + 1.5	+ 10.5	+ 19.5	+ 28.5
Maximum,		+ 20.5	− 43.8	− 25.7	− 9.4	....	....	....
Minimum,		+ 6.0	− 11.1	− 3.0	− 3.0	....	....	....

could take either tension or compression. It is now required to find the actual maximum and minimum stresses due to the combined loads under the limitation that the diagonals can take only tension. To avoid repetition only those members are referred to in the following explanation whose treatment differs from that of the preceding article.

The minimum stress in *GH* is zero as the compression due to the live panel loads at 1, 2, and 3 is greater than its dead load tension. The maximum stress in the counter *JK* is + 14.3 − 5.9 = + 8.4 tons; the minimum stress is zero since the dead load as well as the live panel loads at 1, 2, 3, and 4 tend to compress this member. A counter is therefore required in the fourth and fifth panels of the truss. The counters *LM* and *NO* are not theoretically required because the greatest tensile stress produced by the live load in each one is not sufficient to overcome the tendency of the dead load to compress the same. This is seen also from the fact that the minimum stresses in *CD* and *EF* are + 26.9 and + 10.4 tons respectively, which

implies that their dead load tension is not reduced to zero under the most unfavorable position of the live load.

In finding the maximum and minimum stresses in any vertical it is necessary to consider whether the adjacent diagonals shown in the truss diagram really act under the various conditions of loading. If it is found that they do not act, then the stresses given by the table for the vertical cannot occur.

The live panel loads at 3, 4, 5, 6, and 7 together with the dead load produce in *DE* the maximum compressive stress equal to $-27.3 - 16.5 = -43.8$ tons, provided the adjacent diagonals are in tension. Under the influence of these loads *CD* and *EF* are both found to be in tension, and hence the value just obtained is the required stress. The live loads at 1 and 2 acting in addition to the dead load produce the minimum stress of $+5.4 - 16.5 = -11.1$ tons for the same reason.

The minimum stress in *FG* is due to the live panel loads at 1, 2, and 3, which with the dead load give a stress of $+10.9 - 7.5 = +3.4$ tons, provided the adjacent diagonals *EF* and *GH* are in tension. These loads would cause a tension of 22.3 tons in *EF* and 8.4 tons compression in *GH*, but as *GH* cannot take compression the counter-tie in the same panel is brought into action. The stress of $+3.4$ tons in *FG* therefore cannot occur. When the main tie acts on its left and the counter on its right, the vertical simply supports the dead panel load on the top chord, hence the minimum stress in *FG* is -3.0 tons. In the same way the minimum stress in *HJ* is also found to be -3.0 tons. Under the live panel loads at 5, 6, and 7 with the dead load the diagonals *GH* and *JK* are strained, hence the stress of -10.9 tons in *HJ* must be added to that produced by the dead load, $+1.5$ tons, to give the maximum of -9.4 tons.

Passing to the verticals on the right of the center it is seen that the combination of loads which would give either the maximum or the minimum stress according to the table will not produce tension in both of the adjacent diagonals, and accordingly no additional values can be inserted in the table. The maximum and minimum stresses for these verticals will be the same as for those on the left of the center. The tabulation for the verticals beyond *HJ* is also shown to be unnecessary as *KL* in Fig. 59 has no diagonal on its right, the counters *LM* and *NO* not being required.

The chord stresses are found in the same way as for dead load (Art. 28), only the main ties however being inserted in the truss diagram. The stresses in a Howe truss are determined in a similar manner to that employed for the Pratt truss, the diagonals in that case taking only compression.

The following important principles may now be stated, attention to which will materially reduce the work in solving other problems for either type of truss mentioned:

The maximum stress in any vertical or main diagonal is produced when the live load covers the larger segment of the span.

The maximum stress in any counter diagonal occurs when the live load covers the smaller segment of the span.

The minimum stresses in both diagonals of a counter-braced panel are zero.

The minimum stress in the diagonal of a panel not counter-braced is given when the live load covers the smaller segment of the span.

The minimum stress in any vertical adjacent to a counter diagonal equals the dead apex load on the upper chord of a Pratt truss or the lower chord of a Howe truss.

The minimum stress in a vertical not adjacent to a counter diagonal is produced when the live load covers the smaller segment of the span.

Prob. 42. A Howe truss of 12 panels for a through single track railroad bridge has a span of 123 feet and a depth of 15

feet. The dead load is 625 pounds per linear foot, one-third to be taken on the upper chord, and the uniform train load is 1 700 pounds per linear foot per truss. Find the maximum and minimum stresses.

ART. 31. SNOW LOAD STRESSES.

In addition to the dead and live load stresses must be considered those due to the snow and wind. The snow load for highway bridges is taken lower than for roofs since in the country it is not probable that the full live load would come on the bridge while a heavy fall of snow rests upon it, while in towns the sidewalks are generally cleared of snow. The snow load may vary from 20 to 0 pounds per square foot of floor surface depending upon the climate where the bridge is situated. As the floor of railroad bridges is open so that but little is retained no snow load is regarded.

As the snow load is uniform the stress diagram is exactly similar to that for dead load if the latter be taken only on the chord supporting the floor, or, like the diagram for a full live load. A separate diagram is hence not required as the stresses may be obtained from either of those mentioned by graphic multiplication, the same ratio existing between the stresses as that of the respective panel loads.

For the example in the preceding article the snow panel load is

$$\tfrac{1}{2} \times 21 \times 22 \times 15 = 3\,465 \text{ pounds} = 1.73 \text{ tons},$$

for a load of 15 pounds per square foot of floor surface for the roadway only. The ratio of snow to uniform live load stresses is therefore $\frac{1.73}{14.52} = 0.119$. This gives a snow load stress of -8.0 tons in the end post AB, $+3.4$ tons in EF, -2.6 tons in DE, etc.

Prob. 43. A through Pratt truss for a highway bridge in a village has 12 panels each 11 feet long and 14 feet deep. The roadway is 18 feet 9 inches wide, and there are two sidewalks each 5 feet wide. Find the stresses due to a snow load of 10 pounds per square foot.

ART. 32. WIND STRESSES.

The greatest stresses due to wind are produced when it blows horizontally at right angles to the line of the bridge. The surface exposed to wind action is usually taken as double the area of the side elevation of one truss. If this area be not known an approximate value may be obtained by taking as many square feet as there are linear feet in the skeleton outline of the truss. For railroad bridges the surface of the side of a train, taken at 10 square feet per linear foot of train, is added to the above. No similar addition is made for highway bridges as it is not probable that the live loads would cover them when the wind is blowing at its maximum rate.

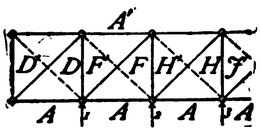


Fig. 63.

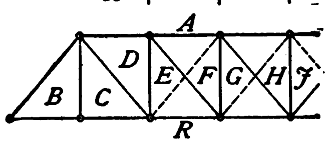


Fig. 64.

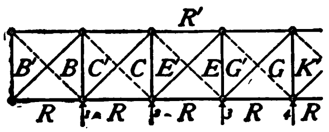


Fig. 65.

The wind pressure is taken from 30 to 40 pounds per square foot and produces its maximum effect when acting like a live load. The wind load on the trusses is divided between the upper and lower lateral bracing while the wind load upon the train is all taken by the lateral bracing of those chords which support the floor. The lateral bracing is generally of the Pratt type, the floor beams acting as the normal struts in one of the systems.

For an example let the through Pratt truss highway bridge whose dimensions are given in Art. 28 be again taken. The

side elevation of the outline of the left half of the truss is shown in Fig. 64, the plan of the upper lateral bracing in Fig. 63, and that of the lower lateral bracing in Fig. 65. When the wind blows in the direction indicated by the arrows and moves from the right toward the left the diagonals drawn in full lines are strained and when it blows in the opposite direction the other set of diagonals is strained.

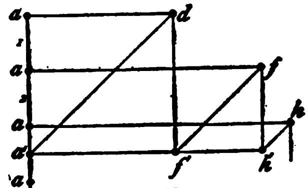


Fig. 66.

The approximate area exposed to wind action is,

$$176 + 132 + 7 \times 26 + 12 \times 34.1 = 889 \text{ square feet.}$$

Taking the wind pressure at 40 pounds per square foot the total wind load is

$$889 \times 40 = 35\,560 \text{ pounds} = 17.78 \text{ tons,}$$

and the wind panel load is

$$17.78 \div (6 + 8) = 1.27 \text{ tons.}$$

The chord stresses are determined for uniform wind load by means of Fig. 66 for the upper lateral bracing and from Fig. 67 for the lower system. Only one-half of each diagram is shown, the other half being symmetrical with it. When the wind blows in the opposite direction the chord stresses on each side of the bridge will exchange values.

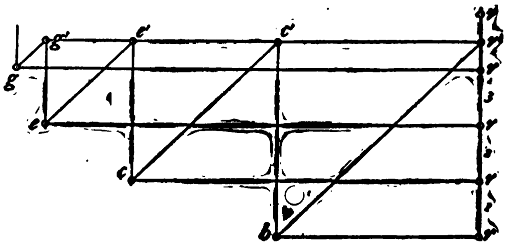


Fig. 67.

For the wind blowing in the direction of the arrows the diagrams give the following stresses in tons; for the upper chord,

AD	*AF*	*AH*	*A'F'*	*A'H'*
− 3.3	− 5.3	− 6.0	+ 3.3	+ 5.3

side elevation of the outline of the left half of the truss is shown in Fig. 64, the plan of the upper lateral bracing in Fig. 63, and that of the lower lateral bracing in Fig. 65. When the wind blows in the direction indicated by the arrows and moves from the right toward the left the diagonals drawn in full lines are strained and when it blows in the opposite direction the other set of diagonals is strained.

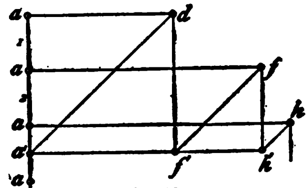


Fig. 66.

The approximate area exposed to wind action is,

$$176 + 132 + 7 \times 26 + 12 \times 34.1 = 889 \text{ square feet.}$$

Taking the wind pressure at 40 pounds per square foot the total wind load is

$$889 \times 40 = 35\,560 \text{ pounds} = 17.78 \text{ tons,}$$

and the wind panel load is

$$17.78 \div (6 + 8) = 1.27 \text{ tons.}$$

The chord stresses are determined for uniform wind load by means of Fig. 66 for the upper lateral bracing and from Fig. 67 for the lower system. Only one-half of each diagram is shown, the other half being symmetrical with it. When the wind blows in the opposite direction the chord stresses on each side of the bridge will exchange values.

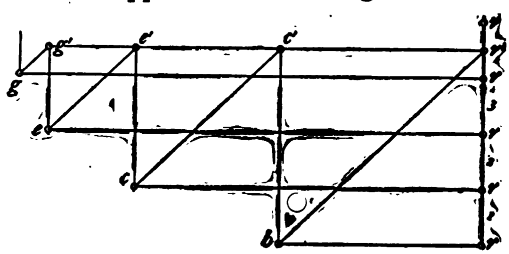


Fig. 67.

For the wind blowing in the direction of the arrows the diagrams give the following stresses in tons; for the upper chord,

AD	AF	AH	$A'F'$	$A'H'$
− 3.3	− 5.3	− 6.0	+ 3.3	+ 5.3

and for the lower chord,

RB	*RC*	*RE*	*RG*	*R'C'*	*R'E'*	*R'G'*
− 4.7	− 7.9	− 9.9	− 10.6	+ 4.7	+ 7.9	+ 9.9

In accordance with the simplified construction given in Art. 30 let a horizontal and a vertical line be drawn through *s* in Fig. 68. Let *st* be laid off equal to 21 feet and *su* equal to 22 feet, then *tus* will be the angle which the diagonals make with the chords. With a suitable scale of force let *sw* be made equal to 1.27 tons and *wx* drawn parallel to *tu*. Applying the scale to *wx* it is found to measure 1.84 tons. Dividing this value by 6 and 8, the number of panels in the upper and lower chords respectively, the quotients 0.31 and 0.23 tons are obtained which form the basis of the tabulations for the stresses in the diagonals. Dividing the panel load, 1.27 tons, by the same numbers the corresponding quotients 0.21 and 0.16 tons are found for the verticals. The following tables are now prepared:

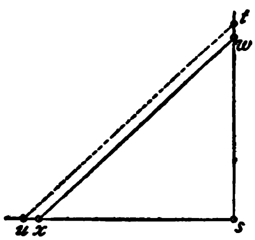


Fig. 68.

For the upper lateral bracing,

	Diagonals.			Struts.		
	DD'	*FF'*	*HH'*	*DF'*	*FH'*	*HJ'*
Wind panel load at 1	+ 1.55	....	....	− 1.05	....	....
2	+ 1.24	+ 1.24	....	− 0.84	− 0.84	....
3	+ 0.93	+ 0.93	+ 0.93	− 0.63	− 0.63	− 0.63
4	+ 0.62	+ 0.62	+ 0.62	− 0.42	− 0.42	− 0.42
5	+ 0.31	+ 0.31	+ 0.31	− 0.21	− 0.21	− 0.21
Maximum wind stresses	+ 4.7	+ 3.1	+ 1.9	− 3.2	− 2.1	− 1.3

and for the lower lateral bracing,

	Diagonals.				Struts.			
	BB'	*CC'*	*EE'*	*GG'*	*BC'*	*CE'*	*EG'*	*GK'*
Wind panel load at 1	+ 1 61	....	....	....	− 1.12	....	....	....
2	+ 1.38	+ 1.38	....	....	− 0.96	− 0.96	....	....
3	+ 1.15	+ 1.15	+ 1.15	....	− 0.80	− 0.80	− 0.80	....
4	+ 0.92	+ 0.92	+ 0.92	+ 0.92	− 0.64	− 0.64	− 0.64	− 0.64
5	+ 0.69	+ 0.69	+ 0.69	+ 0.69	− 0.48	− 0.48	− 0.48	− 0.48
6	+ 0.46	+ 0.46	+ 0.46	+ 0.46	− 0.32	− 0.32	− 0.32	− 0.32
7	+ 0.23	+ 0.23	+ 0.23	+ 0.23	− 0.16	− 0.16	− 0.16	− 0.16
Maximum wind stresses	+ 6.4	+ 4.8	+ 3.5	+ 2.3	− 4.5	− 3.4	− 2.4	− 1.6

Both diagonals in the same panel have equal stresses due to wind. The minimum stresses in all the web members are zero. Since every panel is counter-braced only tensile stresses in the diagonals and compressive stresses in the struts need to be tabulated.

Prob. 44. A through single track railroad bridge has a span of 120 feet. Its trusses are of the Pratt type, have 6 panels, 21 feet deep, and are 16 feet apart between centers. Find the stresses due to a wind pressure of 40 pounds per square foot, provided only the wind pressure on the train be considered as a moving load.

ART. 33. STRESSES DUE TO INITIAL TENSION.

In trusses whose diagonals take only tension the counter-ties are made adjustable in order to be drawn up to a certain degree of tension when the bridge is unloaded. The stress thus introduced in these truss members is called initial tension, and serves to prevent the vibration of the diagonals in the counter-braced panels under moving loads and to stiffen the truss as a whole.

It is required to determine the stresses produced in other members of the truss when all the counters are subjected to a given amount of initial tension. The number of counters in practice is larger than is theoretically required. As the stress in any counter is equivalent to two external forces, each equal to the initial tension, applied at the joints united by the counter-tie and acting toward each other, it may be replaced by them in this analysis.

In the Pratt truss considered in Arts. 28, 30, and 32 let the counters in the third, fourth, fifth, and sixth panels be each subject to an initial tension of 4 tons. The truss diagram is shown in Fig. 69, each of the external forces being equal to 4 tons.

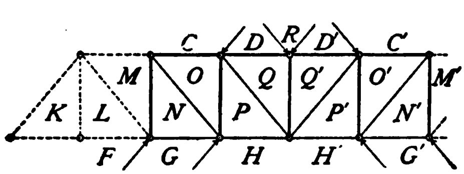


Fig. 69.

In Fig. 70 let these external forces, taken in regular order around the truss, be laid off, thus forming the closed polygon *cdrd'c'g'h'hgc*. Since each pair of forces is in equilibrium the entire system is in equilibrium and hence there are no reactions at the supports. No forces being applied at any of the joints of the first panel the members drawn in broken lines may be omitted. The stress diagram is now completed in the usual way, the characters of the stresses determined, and their magnitudes found by applying the scale, the results, expressed in tons, being as follows:

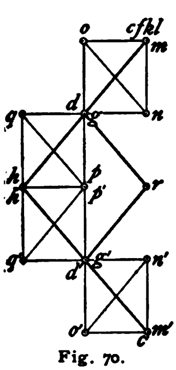


Fig. 70.

CHORDS.	STRESSES.	VERTICALS.	STRESSES.	MAIN TIES.	STRESSES.
CO and *GN*	−2.6	*MN*	−3.1	*NO*	+4.0
DQ and *HP*	−2.6	*OP* and *QQ'*	−6.1	*PQ*	+4.0

The stress diagram shows that the tension in any counter affects only the members of the panel to which it belongs. It produces compression in both chords equal to the horizontal

component of the initial tension, compression in the verticals equal to its vertical component, and tension in the main tie equal to that in the counter. The general effect is therefore to increase the stresses due to other loads in all the members of the counter-braced panels except the lower chord, whose stresses are diminished. In the upper and lower lateral bracing both diagonals in each panel are made adjustable.

It is not necessary to make stress diagrams for these lateral systems, but simply to draw a right triangle whose base is parallel to the chords and whose hypothenuse, measuring 4.0 tons, is parallel to one of the diagonals. Applying the scale the base is found to measure 2.9 tons and the perpendicular 2.8 tons. The stress in the chords is therefore -2.9 tons throughout; in the end struts -2.8 tons; in the remaining struts $2(-2.8) = -5.6$ tons; and in the diagonals $+4.0$ tons. When the struts are not normal to the chords it is best to construct the complete stress diagrams.

It has not been customary, however, to consider the stresses caused by initial tension in the counters except those in the main ties. An examination of the tables in the next article will show the relation which these stresses bear to the others and to the final maximum and minimum stresses.

Prob. 45. A through double-track railroad bridge 140 feet in span has Pratt trusses of 7 panels and 32 feet deep. The bridge is 28 feet wide between centers of chords. Find the stresses due to an initial tension of 5 tons in every counter of the trusses and lateral systems.

ART. 34. FINAL MAXIMUM AND MINIMUM STRESSES.

The final maximum and minimum stresses in any truss member are the extreme limits of stress to which it is subjected by all possible combinations of the dead, live, snow, and wind loads, and initial tension. The larger limit is called the maximum and the smaller the minimum stress, and they may have the same or opposite signs. In finding the maximum and minimum stresses in the following tables, it is assumed that the initial tension as well as the dead load is always acting. In practice the wind stresses in the chords are more frequently disregarded than taken into account.

In Art. 30 the maximum and minimum stresses due only to dead and live loads were found for the through Pratt truss whose stresses due to snow load were found in Art. 31, those due to wind load in Art. 32, and to initial tension in Art. 33. The various results are now brought together in the following table and the final maximum and minimum stresses obtained by addition. The members are designated as in Figs. 63, 64, and 65.

	UPPER CHORD.			LOWER CHORD.			
	AD	*AF*	*AH*	*RB*	*RC*	*RE*	*RG*
Dead load,	− 45.4	− 56.8	− 60.6	+ 26.5	+ 26.5	+ 45.4	+ 56.8
Live load,	− 73.7	− 92.2	− 98.3	+ 43.0	+ 43.0	+ 73.7	+ 92.2
Snow load,	− 8.8	− 11.0	− 11.7	+ 5.1	+ 5.1	+ 8.8	+ 11.0
North wind,	− 3.3	− 5.3	− 6.0	− 4.7	− 7.9	− 9.9	− 10.6
South wind,	0	+ 3.3	+ 5.3	0	+ 4.7	+ 7.9	+ 9.9
Initial tension—Truss,	0	− 2.6	− 2.6	0	0	− 2.6	− 2.6
" —Lateral system,	− 2.9	− 2.9	− 2.9	− 2.9	− 2.9	− 2.9	− 2.9
Maximum stress,	− 134.1	− 170.8	− 182.1	+ 71.7	+ 76.4	+ 130.3	+ 164.4
Minimum stress,	− 48.3	− 59.0	− 60.8	+ 18.9	+ 15.7	+ 30.0	+ 40.7

	END POST.	MAIN TIES.			COUNTER-TIE.	VERTICALS.			
	AB	*CD*	*EF*	*GH*	*GH*	*BC*	*DE*	*FG*	*HJ*
Dead and live load max.,	− 107.8	+ 79.3	+ 53.3	+ 29.7	+ 8.4	+ 20.5	− 43.8	− 25.7	− 9.4
Dead and live load min.,	− 41.1	+ 26.9	+ 10.4	0	0	+ 6.0	− 11.1	− 3.0	− 3.0
Snow load,	− 8.0	+ 5.7	+ 3.4	+ 1.1	0	+ 1.7	− 2.6	− 0.9	0
Initial tension,	0	0	+ 5.0	+ 5.0	+ 5.0	0	− 3.1	− 6.1	− 6.1
Maximum stress,	− 115.8	+ 85.0	+ 61.7	+ 35.8	+ 13.4	+ 22.2	− 49.5	− 32.7	− 15.5
Minimum stress,	− 41.1	+ 26.9	+ 15.4	+ 5.0	+ 5.0	+ 6.0	− 14.2	− 9.1	− 9.1

	Upper Lateral Bracing.					
	Diagonals.			Struts.		
	DD'	*FF'*	*HH'*	*DF'*	*FH'*	*HJ'*
Wind,	+ 4.7	+ 3.1	+ 1.9	− 3.2	− 2.1	− 1.3
Initial tension,	+ 4.0	+ 4.0	+ 4.0	− 5.6	− 5.6	− 5.6
Maximum stress,	+ 8.7	+ 7.1	+ 5.9	− 8.8	− 7.8	− 6.9
Minimum stress,	+ 4.0	+ 4.0	+ 4.0	− 5.6	− 5.6	− 5.6

	Lower Lateral Bracing.							
	Diagonals.				Struts.			
	BB'	*CC'*	*EE'*	*GG'*	*BC'*	*CE'*	*EG'*	*GK'*
Wind,	+ 6.4	+ 4.8	+ 3.5	+ 2.3	− 4.5	− 3.4	− 2.4	− 1.6
Initial tension,	+ 4.0	+ 4.0	+ 4.0	+ 4.0	− 5.6	− 5.6	− 5.6	− 5.6
Maximum stress,	+ 10.4	+ 8.8	+ 7.5	+ 6.3	− 10.1	− 9.0	− 8.0	− 7.2
Minimum stress,	+ 4.0	+ 4.0	+ 4.0	+ 4.0	− 5.6	− 5.6	− 5.6	− 5.6

Prob. 46. Find the maximum and minimum stresses in the chords of the above example, provided the effect of the wind be disregarded. Also, compute the greatest percentage of reduction in the maximum stress of any chord member on this account.

ART. 35. THE BOWSTRING TRUSS.

This form of truss is shown in Figs. 71, 72, and 73, and is frequently used for highway bridges. The apex points of the upper chord lie upon the arc of a circle. When the bracing is arranged like that in Fig. 71, the diagonals take only tension, while the verticals take either tension or compression. In the truss in Fig. 73, all the web members are made to sustain either kind of stress. The same is true of the form given in Fig. 72, with the exception of the middle and end verticals, which are subject to tension only.

For example, let a truss like Fig. 71 be taken whose upper panel points lie in the arc of a circle. Let it have 8 panels, each 14 feet long on the lower chord, with a depth at the center of 16 feet. The bridge has a roadway 22 feet wide and two sidewalks each 5 feet wide.

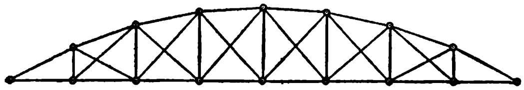


Fig. 71.

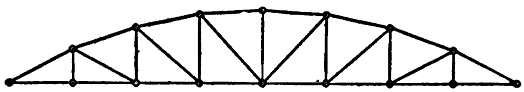


Fig. 72.

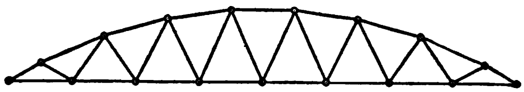


Fig. 73.

The dead panel load is found to be 4.19 tons, of which 1.40 tons is to be taken on the upper chord and 2.79 tons on the

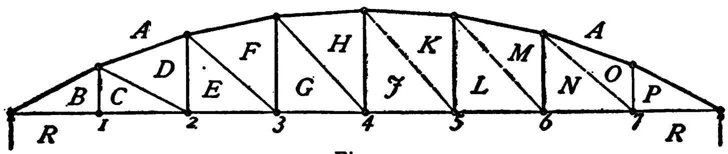


Fig. 74.

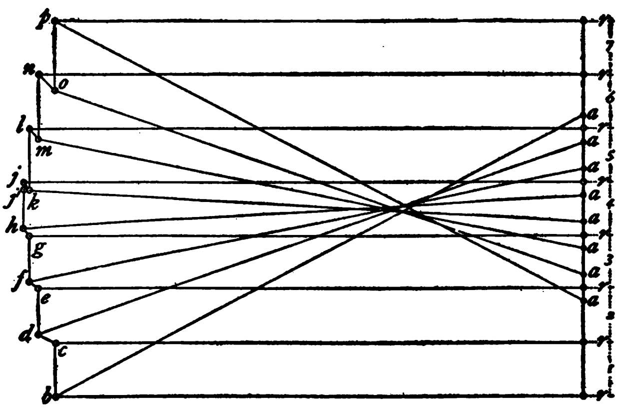


Fig. 75.

lower. The snow panel load is 1.68 tons. At 90 pounds per square foot of floor surface the live panel load is 10.08 tons, or 6 times the snow load.

Let a truss diagram be drawn as in Fig. 74, containing only the main diagonals in the left half and the counters in the right. The depths of the truss at the first, second, and third panel points are 7.32, 12.23, and 15.07 feet respectively. The stress diagram obtained for dead load is shown in Fig. 75, that for a live panel load at apex 1 in Fig. 76, that for a live panel load at apex 7 in Fig. 77, and that for a uniform live load in Fig. 78. As a check upon the construction of these diagrams it is observed that in Figs. 75 and 78 *bc* and *op* are in the same vertical line. The same is true of *de* and *mn* and of *fg* and *kl*. In general the lines representing stresses in verticals equally distant from the center of the truss lie in the same vertical line, or are equally distant from the load line. In Fig. 76 the line *de* is at the same distance from the load line as *nm* in Fig. 77, also *nm* in Fig. 76 and *ed* in Fig. 77 are similarly situated. The same relation exists between the lines representing stresses in any two verticals occupying symmetri-

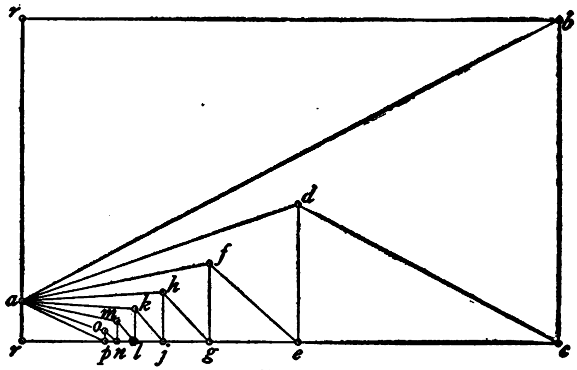


Fig. 76.

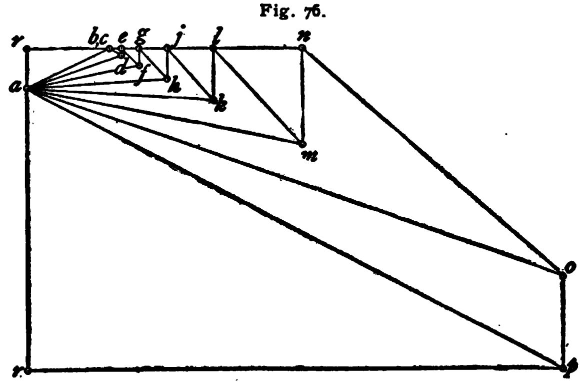


Fig. 77.

cal positions in the truss. Again, if in Fig. 77 *lk* be produced to meet *am* and the intersection be called *k'*, then *lk'* will be equal to *fg* in Fig. 76 and *nk'* will be equal to *fe* in Fig. 76.

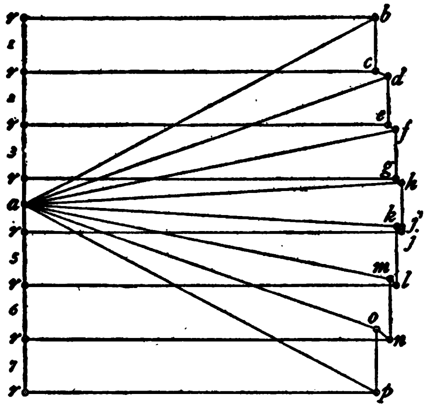


Fig. 78.

In Figs. 75 and 78 *hj'* represents the stress in *HJ* when the main diagonal is inserted on its right instead of the counter shown in Fig. 74, the point *j'* being at the intersection of the lines *ak* and *hj*. Only the stresses in web members are scaled off from Figs. 76 and 77. The snow load stresses are obtained by dividing those due to uniform live load by six.

The stress diagrams from which the following results were obtained were drawn to the following scales: The dead load diagram, 4 tons to an inch; the diagrams for single live panel loads, 2 tons to an inch; and the uniform live load diagram, 10 tons to an inch.

The results expressed in tons are now tabulated as in Art. 30 and the maximum and minimum stresses obtained, the effect of wind and initial tension being omitted.

	UPPER CHORD.				LOWER CHORD.		
	AB	*AD*	*AF*	*AH*	*RB = RC*	*RE*	*RG*
Dead Load,	− 31.6	− 30.5	− 29.8	− 29.4	+ 28.0	+ 28.8	+ 29.2
Live Load,	− 76.1	− 73.2	− 71.5	− 70.6	+ 67.4	+ 69.1	+ 70.1
Snow Load,	− 12.7	− 12.2	− 11.9	− 11.8	+ 11.2	+ 11.5	+ 11.7
Maximum,	− 120.4	− 115.9	− 113.2	− 111.8	+ 106.6	+ 109.4	+ 111.0
Minimum,	− 31.6	− 30.5	− 29.8	− 29.4	+ 28.0	+ 28.8	+ 29.2

		Main Diagonals.			Counters.		
		CD	*EF*	*GH*	*JK*	*LM*	*NO*
Live panel load at apex	1	− 9.3	− 3.8	− 2.1	− 1.4	− 0.9	− 0.6
	2	+ 3.2	− 7.5	− 4.2	− 2.7	− 1.8	− 1.3
	3	+ 2.7	+ 4.2	− 6.4	− 4.1	− 2.8	− 1.9
	4	+ 2.1	+ 3.3	+ 5.2	− 5.4	− 3.7	− 2.5
	5	+ 1.6	+ 2.5	+ 3.9	+ 6.6	− 4.6	− 3.2
	6	+ 1.1	+ 1.7	+ 2.6	+ 4.4	+ 8.3	− 3.8
	7	+ 0.5	+ 0.8	+ 1.3	+ 2.2	+ 4.1	+ 10.9
+ Total,		+ 11.2	+ 12.5	+ 13.0	+ 13.2	+ 12.4	+ 10.9
− Total,		− 9.3	− 11.3	− 12.7	− 13.6	− 13.8	− 13.3
Uniform live load,		+ 1.9	+ 1.2	+ 0.3	− 0.4	− 1.4	− 2.4
Dead load,		+ 0.8	+ 0.6	+ 0.2	− 0.2	− 0.6	− 1.0
Snow load,		+ 0.3	+ 0.2	+ 0.1	− 0.1	− 0.2	− 0.4
Maximum,		+ 12.3	+ 13.3	+ 13.3	+ 13.0	+ 11.8	+ 9.9
Minimum,		0	0	0	0	0	0

		Verticals.						
		BC	*DE*	*FG*	*HJ*	*KL*	*MN*	*OP*
Live panel load at apex	1	+ 10.1	+ 4.3	+ 2.5	+ 1.6	+ 1.0	+ 0.7	+ 0.4
	2	0	+ 8.6	+ 4.9	+ 3.1	+ 2.0	+ 1.3	+ 0.8
	3	0	− 1.2	+ 7.3	+ 4.6	+ 3.1	+ 2.0	+ 1.2
	4	0	− 1.0	− 2.2	+ 6.2	+ 4.1	+ 2.7	+ 1.6
	5	0	− 0.7	− 1.6	− 2.9	+ 5.1	+ 3.4	+ 2.1
	6	0	− 0.5	− 1.1	− 1.9	− 3.3	+ 4.0	+ 2.5
	7	0	− 0.2	− 0.5	− 1.0	− 1.6	− 3.0	+ 2.9
+ Total,		+ 10.1	+ 12.9	+ 14.7	+ 15.5	+ 15.3	+ 14.1	+ 11.5
− Total,		0	− 3.6	− 5.4	− 5.8 [+ 9.3]	− 4.9	− 3.0	0
Uniform live load,		+ 10.1	+ 9.3	+ 9.3	+ 9.7 [+ 2.5]	+ 10.4	+ 11.1	+ 11.5
Dead load,		+ 2.8	+ 2.4	+ 2.4	+ 2.6 [+ 1.5]	+ 3.0	+ 3.3	+ 3.4
Snow load,		+ 1.7	+ 1.5	+ 1.5	+ 1.6	+ 1.7	+ 1.9	+ 1.9
Maximum,		+ 14.6	+ 13.2	+ 13.2	+ 13.3	....	....	....
Minimum,		+ 2.8	− 1.2	− 3.0	− 3.2	− 1.9	+ 0.3	....

Counters are required in every panel of the truss. The table also shows that the greatest tension in any vertical due to live load requires the panel points on the left to be loaded, but this stress cannot occur since the adjacent diagonals shown in Fig. 74 are not in tension under those loads. The values in the

Prob. 47. A through bowstring truss has six panels each 15 feet long, the depth at the first and fifth panel points being 7.5 feet, at the second and fourth panel points 11.7 feet, and at the center 13 feet. The dead panel load is 2.5 tons and the live panel load 7.5 tons. Find the maximum and minimum stresses due to these loads only.

ART. 36. THE PARABOLIC BOWSTRING TRUSS.

When the panel points of the broken chord of a bowstring truss lie upon a parabola whose vertex is midway between the supports, the stress diagrams become simpler. Let a parabolic bowstring truss be taken with the same general dimensions and loads as given in the preceding article. In the diagrams like Figs. 75 and 78 the broken lines *bcd* . . . *nop* become straight lines, and the points *c* and *d*, *e* and *f*, . . . *n* and *o*, coincide. This shows that under a uniform load the stress in the horizontal chord is the same throughout, the diagonals are not strained at all, and each vertical carries only the panel load on the horizontal chord. In the diagrams similar to Figs. 76 and 77, the points *b*, *d*, *f*, *h*, *k*, *m*, and *o* lie upon a straight line which intersects the load line produced at the shorter distance *ar* from *r*, thus checking the construction.

In the tabulated stresses for the webbing the sum of the '+ total' and the '— total' will give zero for the diagonals and + 10.08 for the verticals provided the work be done with the utmost precision. With diagrams like Figs. 76 and 77 made to a scale of 3 tons to an inch, the stresses obtained by tabulation for uniform live load averaged 0.05 tons in magnitude for the diagonals, three being tension and three compression, and those in the verticals varied on an average 0.01 tons from the true result, some being too large and others too small.

The final results in tons are given in the following table:

CHORDS.	MAXIMUM STRESSES.	MINIMUM STRESSES.	DIAGONALS.	MAXIMUM STRESSES.	VERTICALS.	MINIMUM STRESSES.
AB	− 124.9	− 32.8	*CD*	+ 9.9	*BC*	+ 2.8
AD	− 118.6	− 31.2	*EF*	+ 11.6	*DE*	− 0.4
AF	− 114.1	− 30.0	*GH*	+ 12.9	*FG*	− 2.3
AH	− 111.9	− 29.4	*JK*	+ 13.4	*HJ*	− 2.9
					KL	− 2.3
RB, *RC*, *RE*, *RG*	+ 111.7	+ 29.3	*LM*	+ 12.9	*MN*	− 0.4
			NO	+ 11.6	*OP*	− 2.8

The minimum stresses in the diagonals are zero, and the maximum stress in each vertical equals

$$2.79 + 10.08 + 1.68 = 14.55 \text{ tons},$$

or the sum of the dead, live, and snow panel loads.

If this truss were used as a deck bridge the maximum stresses in the verticals would be those given in the accompanying table, while the minimum stresses would equal the dead panel load on the horizontal chord, or − 2.79 tons. It will be observed that the differences between the minimum stresses in the verticals of the through truss and the maximum stresses in the same members of the deck truss equal twice the dead panel load plus the live and snow panel loads. The stresses in the remaining members are the same for a deck as for a through bridge except that the chord stresses change in character.

VERTICALS.	MAXIMUM STRESSES.
BC and *OP*	− 14.5
DE and *MN*	− 17.7
FG and *KL*	− 19.6
HJ	− 20.2

The properties of the parabola are such as to provide a very simple and abridged construction for obtaining directly the maximum and minimum stresses due to dead, live, and snow loads.

Let S be the stress in the horizontal chord due to the total uniform load W (including the half panel loads at the supports), l the span and d the depth of the truss at the center, then (Roofs and Bridges, Part I, Art. 39),

$$S = \frac{Wl}{8d}.$$

Substituting for these terms their values for the truss considered above, the stress due to live load is

$$S = \frac{8 \times 10.08 \times 112}{8 \times 16} = 70.56 \text{ tons.}$$

Similarly the stress due to dead load is 29.33 tons and that due to snow load is 11.76 tons.

Now in Fig. 80 on the horizontal line *ad* with a scale of 4 or 5 tons to an inch, let *ab* be laid off equal to 29.33 tons, *bc* equal to

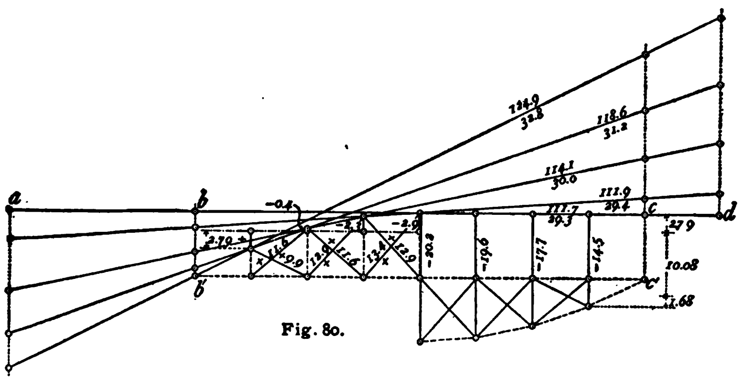


Fig. 80.

70.56 tons, *cd* equal to 11.76 tons, and verticals erected at each point of division. As the depth of the truss is one-seventh of its span let *bb'* and *cc'* be made equal to one-seventh of 70.56 or 10.08 tons, and on the span *b'c'* let an outline diagram be drawn similar to the truss diagram. In the figure one-half is drawn as a through and the other half as a deck truss. By measuring the diagonals with the scale of force their maximum stresses are obtained. Their minimum stresses are zero.

Let the chord members be prolonged until they meet the verticals through *a* and *d*. Each of these lines is divided into three parts by the four verticals, these parts giving the stresses due to dead, live, and snow load respectively. For example, the dead load stress in the horizontal chord is represented by the part *ab*, the live load stress by *bc*, and the snow load stress by *cd*. The maximum stress in the same member is hence *ad*, or 111.7 tons, and the minimum stress is *ab*, or 29.3 tons.

On the through truss draw the horizontal broken and dotted base line 2.79 tons (the value of the dead panel load) above the first panel point on the upper chord. By measuring the verticals extending from this base line to each panel point on the upper chord, upward being compression and downward tension, the minimum stresses in the verticals are found. Their maximum stresses are each equal to the sum of the dead, live, and snow panel loads, or 14.55 tons.

On the deck truss let a similar base line be drawn 14.55 tons above the first panel point from the support on the lower chord, and the verticals measured from the panel points to the base line; thus are found the maximum stresses in the verticals, all of them being compression. Their minimum stresses are each equal to — 2.79 tons. The measured stresses are marked on the different lines of the diagram.

Prob. 48. A deck parabolic bowstring truss of 10 panels has a span of 120 feet and a depth of 15 feet at the center. Find the maximum and minimum stresses for a dead panel load of 3 tons and a live panel load of 7 tons.

ART. 37. APPLICATION OF THE EQUILIBRIUM POLYGON.

In the preceding articles of this chapter the method of the force polygon has been employed exclusively. To illustrate the application of the equilibrium polygon in the determination

of stresses let the same example used in Art. 28 be taken, it being required to find the chord stresses, and afterward the web stresses due to the dead load. The span is 176 feet, the depth 26 feet, and the dead panel load 8.95 tons.

Let the truss diagram be drawn to a scale of 10 feet to an inch and the panel loads laid off successively on the load line *qy* in Fig. 81 to a scale of 10 tons to an inch (considerably re-

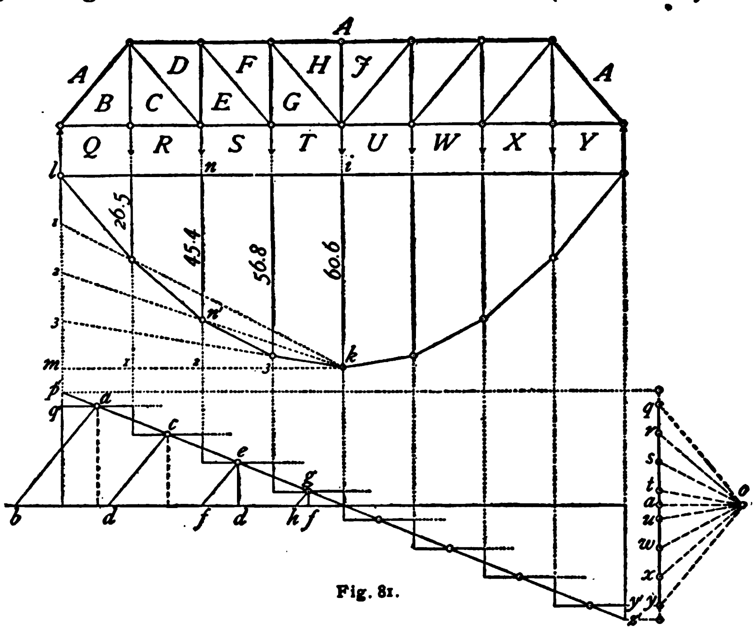


Fig. 81.

duced as here printed). The effective reactions are *ya* and *aq*, the load line being bisected at *a*. Let the pole *o* be taken on a horizontal through *a*, the pole distance *H* be made equal to 26 tons, and the equilibrium polygon constructed (Art. 6). The ordinates at the vertices of this polygon when measured by the linear scale and multiplied by *H* give the bending moments in tons-feet at the corresponding sections of the truss (Art. 7). The chord stresses are obtained by dividing these moments by 26 feet, the depth of the truss. For instance, the ordinate nn' measures 45.4 feet, whence the stress in AD or ES is

$$\frac{45.4 \times 26}{26} = 45.4 \text{ tons.}$$

The stresses may therefore be directly obtained by measuring the ordinates with a scale of 10 tons to an inch, the results being marked on the diagram. Even with a smaller scale than that indicated above the same stresses, measured to tenths of a ton, are obtained as in Art. 28.

If the linear scale be 20 feet to an inch, and H be taken as 52 tons, the scale of tons remaining the same, the ordinates should be measured by a scale of $20 \times \frac{52}{26} = 40$ tons to an inch to obtain the chord stresses. Again, if H be laid off by the linear scale equal to double the depth of the truss, then the ordinates are to be measured by double the scale of force or 20 tons to an inch.

As the vertices of the equilibrium polygon lie upon a parabola whose vertex is at k, the ordinates may be obtained without drawing the equilibrium polygon. The chord stress at the center of a truss uniformly loaded is

$$S = \frac{Wl}{8d},$$

in which W includes the half panel load at each support. Let the middle ordinate ik be made equal to

$$S = \frac{8 \times 8.95 \times 176}{8 \times 26} = 60.6 \text{ tons,}$$

let lm be made equal to ik and divided into the same number of parts as mk, in this case four. Drawing radial lines from k to these points of division their intersections with the corresponding verticals give the required points.

For an odd number of panels in a truss lm should be divided into as many parts as there are panels in the entire truss, only

the alternate points of division from *l* to *m* being used. For a uniform live load the same method may be employed as that here given, or the dead and live loads may be combined in one diagram.

The shear diagram for dead load is shown below the moment diagram in Fig. 81, the ordinates representing the vertical shear being limited by the line forming a series of steps from q' to y'. If the load were not concentrated at the panel points but uniform throughout the ordinates for shear would be measured to the straight line $p'z'$, which intersects the former at the center of each panel. The inclined line is the most convenient to use, but usually it is not employed, as the analytic method, consisting only of a few subtractions or additions, is more rapidly applied. The lines *ab*, *cd*, *ef*, and *gh* are the stresses in the diagonals, and *de* and *fg* in two of the verticals. To avoid confusing the diagram *cd*, *ef* and *fg* are turned toward the left, but have the same inclination as the diagonals of the truss. The stress in *BC* is one panel load, and that in *HJ* is zero. If 2.95 tons of the dead panel load be taken on the upper chord, a compression of that amount is to be added to each of the above stresses in the verticals.

In order to determine the maximum live load shear in any panel another shear diagram is necessary. On the horizontal axis *AG* in Fig. 82 let the positions of the panel loads be marked, and their magnitudes laid off on the load line *ag*. Let the pole *o* be placed in a horizontal line through the beginning of the load line, the pole distance made equal to the span of the truss by the linear scale and the equilibrium polygon $A'B'C'$ .. H' constructed. Now let the span be so placed that its right support shall come at *F*, then every panel point from 3 to 7 inclusive is loaded. By Art. 29 this position of the load gives the maximum live load shear in *EF* or *DE* of Fig. 59. The ordinate $F'F''$ is equal to the reaction of the left support and hence equals the vertical shear in the members named; for, the ordinate being contained between the sixth side of the equilibrium polygon and the first side produced measures the sum of the moments of all the loads between them with reference to the section through F' (Art. 7). Calling the value

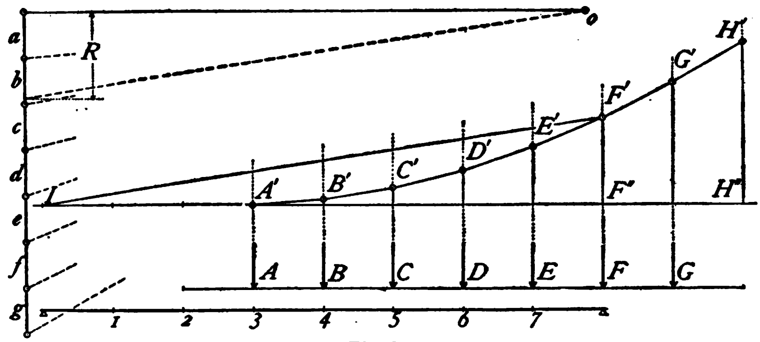


Fig. 82.

of the ordinate y, the sum of these moments equals $y \times H$. But the section at F' is at the right support of the truss, and hence the sum of the moments also equals the moment of the left reaction with reference to this support. Therefore

$$y \times H = R \times l,$$

and since H was made equal to l,

$$y = R.$$

This may also be proved by drawing through the pole a parallel to the closing line $F'I$ of the equilibrium polygon forming a triangle which is equal to $IF'F''$ since one side is equal to its parallel IF'' and all the sides of both triangles are mutually parallel. $F'F''$ is hence equal to its parallel R.

The ordinates taken in succession from H' to A' measured by the scale of force give the maximum live load shears in each panel of the truss from left to right. The stresses in the diagonals are obtained from these shears in the manner indicated in Fig. 81. The results by this method are found to be

the same as those given in Art. 30 in the line '+ total' for the diagonals and in the line '— total' for the verticals.

For trusses with inclined chords the moment diagram gives only the horizontal component of any chord stress, the ordinate being measured in a section passing through the center of moments of the chord member. The shear diagram is not applicable in such cases except for the purpose of finding the reaction from which the stress in the required web member may be found by the method of the force polygon.

It will be observed by the student that the method of the equilibrium polygon does not indicate the character of the stresses as in the method of the force polygon. Whether a web member be in tension or compression is to be determined by cutting it by a plane, noting its direction and the sign of the shear, as was done in the analytic method in Part I, Art. 26.

Prob. 49. Find the maximum and minimum stresses due to dead and live loads for the truss in Prob. 42.

ART. 38. EXCESS LOADS.

It may be specified that a truss shall be designed to carry a given load extending over a certain distance in excess of the uniform live load. In a railroad bridge the excess load would represent the difference between the locomotive panel load and the uniform train panel load.

The chord stress due to a single load P distant x from the left support is a maximum at the load, and for any position of the load

$$S = \frac{M}{d} = \frac{P(l - x)x}{ld};$$

M being the bending moment at the load. If S be an ordinate corresponding to an abscissa x this is the equation of a parabola whose vertex is at the center of the truss. The middle ordinate has a value of $\frac{Pl}{4d}$ and the ordinates are zero at each end. The ordinate at each panel point is the maximum stress in the chord member whose center of moments is at that point.

Although excess loads are not used in determining the stresses in highway bridge trusses, except for the floor system, yet as an illustration of the method let the Pratt truss in Art. 30 be taken and the maximum chord stresses be found for one excess panel load of 5 tons. The ordinates in Fig. 84 are obtained as in the preceding article, the middle ordinate fk being

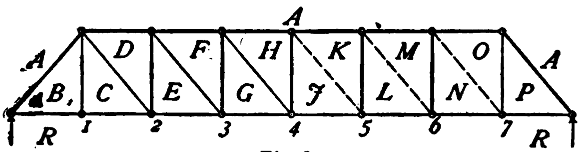


Fig. 83.

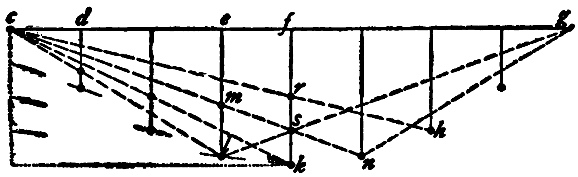


Fig. 84.

$$S = \frac{Pl}{4d} = \frac{5 \times 176}{4 \times 26} = 8.46 \text{ tons.}$$

The other ordinates on either side are 3.7, 6.4, and 7.9 tons.

Let it now be supposed that two excess loads of 5 tons each and 50 feet apart are specified. Let their distance apart be taken as 44 feet or two panel lengths. For two equal loads the maximum stress in a chord member in the left half of the truss occurs when one load is at the section passing through its center of moments and the other load is toward the right. For the member AF one load is to be placed at apex 3 and the other at 5. The stress in AF due to the load at 3 is el which measures 7.9 tons. The stresses for the load at 5 are given by the ordinates of the triangle cng, and since cn intersects el at m, the stress in AF due to this load is em, or 4.8 tons, and that

due to both loads is $7.9 + 4.8 = 12.7$ tons. In a similar manner the other chord stresses are found. The stress in AH may however be larger when the loads are at 3 and 5, than when at 4 and 6, being in this example $6.4 + 6.4 = 12.8$ tons for the former and $8.5 + 4.2 = 12.7$ tons for the latter position, fs being 6.4 tons and fr 4.2 tons.

By inspecting the table in Art. 30 it is seen that the maximum shear in any web member in the left half of the truss due to a single live panel load occurs when the load is placed at the nearest panel point on its right, the shear being equal to the left reaction. For a single excess load P the vertical shear is

$$V = R = \frac{P(l - x)}{l};$$

if V be an ordinate corresponding to an abscissa x this is the equation of a straight line. Thus, when $x = 0$, $V = R$ and when $x = l$, $V = 0$. From these data the diagram for maximum shear due to a rolling load may be readily drawn.

For the above example let the span ab be laid off in Fig. 85, marking the panel points as indicated and erecting verticals at

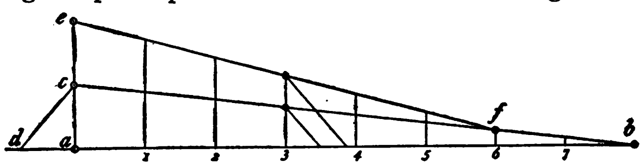


Fig. 85.

all of these points. Let ca be made equal to 5 tons by scale and cb joined, then the ordinates to this line give the maximum shear for each panel. The ordinate at 3 which measures 3.1 tons is the maximum shear in EF and DE and also the stress in DE. The stress in EF is given by a line drawn through the upper extremity of this ordinate with the same slope as EF and is found to be 4.1 tons. The stresses in all the diagonals may be obtained from $cd = 6.56$ tons by the use of a simple ratio which for EF is $\frac{5}{8}$ and for HG is $\frac{1}{8}$. These stresses may also be obtained directly from the tabulation in Art. 30, by multiplying the greatest live load stress in any member due to a single panel load by the ratio of the excess load to the live panel load. For example, the stress in EF is $+11.9 \times \frac{5.0}{14.52} = +4.1$ tons and that in DE is $-9.1 \times \frac{5.0}{14.52} = -3.1$ tons. The same method might be applied to the chords, but would require more work than that used in this article.

For the two excess loads mentioned above the maximum stress in EF or DE is produced by placing one load at 3 and the other at 5. Let ce be drawn equal to $\frac{5}{8} \times 5$ tons and e joined with f which is on the line bc at a distance of two panels from b. The ordinates to the line efb give the maximum shears in each panel due to the two excess loads. The stress in DE is found to be 5.0 tons and in EF 6.6 tons.

The chord stresses are as follows,

	RB and RC	AD and RE	AF and RG	AH
First excess load,	3.7	6.4	7.9	8.5
Addition for second load,	2.6	4.2	4.8	—
Two excess loads,	6.3	10.6	12.7	2 × 6.4 = 12.8

The stresses in the verticals are,

	BC	DE	FG	HJ	KL	MN	OP
First excess load,	+ 5.0	− 3.1	− 2.5	− 1.9	− 1.3	− 0.6	0
Addition for second load,	0	− 1.9	− 1.3	− 0.6	0	0	0
Two excess loads,	+ 5.0	− 5.0	− 3.8	− 2.5	− 1.3	− 0.6	0

and those in the diagonals are,

	AB	CD	EF	GH	JK	LM	NO
First excess load,	− 5.7	+ 4.9	+ 4.1	+ 3.3	+ 2.5	+ 1.6	+ 0.8
Addition for second load,	− 4.1	+ 3.3	+ 2.5	+ 1.6	+ 0.8	0	0
Two excess loads,	− 9.8	+ 8.2	+ 6.6	+ 4.9	+ 3.3	+ 1.6	+ 0.8

These results when combined with the stress given in Art. 34 will increase the maximum stresses in all the members and reduce some of the minimum stresses in the web members. The addition of the stresses in the web members due to excess loads should properly be made in the tables in Art. 30 by inserting a line above the dead load stresses and then obtaining the maximum and minimum stresses anew.

If one or more locomotive panel loads be required to precede the train on a railroad bridge, the maximum chord stresses in the left half of the truss will be obtained by placing the excess loads as near as possible to the left support.

Prob. 50. A Pratt truss for a deck single track railroad bridge has 6 panels each 13 feet 4 inches long and 13 feet 4 inches deep. Find the stresses due to one excess load of 6.5 tons per truss.

ART. 39. LOCOMOTIVE WHEEL LOADS.

A uniform live load which is carried by the stringers and floor beams to the panel points of the trusses, giving uniform live panel loads throughout, is confined mainly to highway

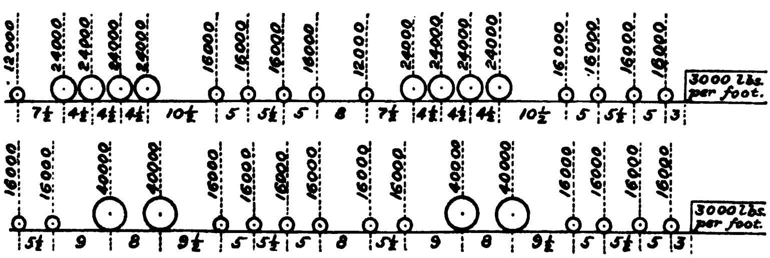


Fig. 86.

bridges. For railroad bridges it is generally specified that the live load shall consist of two coupled locomotives, followed by a uniform train load of a given weight per linear foot of track. The actual wheel loads thus taken constitute a system of concentrated loads whose relations remain unchanged to each other and to the uniform load following them while passing over the bridge.

The first of the diagrams in Fig. 86 represents two typical consolidation locomotives and the second the two typical passenger locomotives and train load specified in 1886 by the Pennsylvania Railroad. The numbers above the wheels show their weights in pounds for both rails of a single track and the numbers between them show their distances apart in feet.

A typical locomotive does not really exist, but is used so that the stresses due to it will be greater than those due to any locomotives that are likely to be built for some years in the future. Different railroads specify various typical locomotives and sometimes actual ones of the heaviest patterns. The typical locomotives shown in Fig. 86 are about the heaviest that have been employed for a number of years in designing

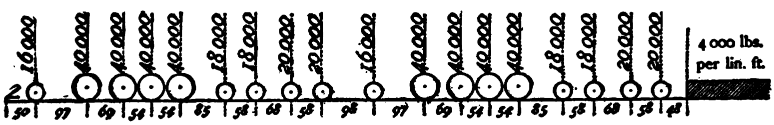


Fig. 87.

railroad bridges, while the two coupled consolidation locomotives and train shown in Fig. 87 have recently been specified by the Lehigh Valley Railroad. The numbers between the wheels in Fig. 87 indicate inches.

Prob. 51. Find the maximum shear in an I beam 15 feet in span under the load represented in Fig. 87.

ART. 40. ANALYSIS OF A PLATE GIRDER.

To illustrate the method of determining the stresses when the live load consists of concentrated wheel loads, let a deck plate girder bridge for a single track railroad be taken, the span being 55 feet, measured between centers of bed plates, and the effective depth 5.5 feet. The total weight of both

girders and of the lateral bracing is 16.45 tons, and the floor system is estimated at 420 pounds per linear foot. The live load is to consist of one Lehigh Valley consolidation locomotive and tender followed by a uniform train load of 4000 pounds per linear foot. It is required to find the maximum flange stresses and the maximum shears throughout the girder due to the above loads.

The total dead load for each girder is first found to be,

$$\frac{1}{2}\left(\frac{420 \times 55}{2000} + 16.45\right) = 14.0 \text{ tons},$$

which is regarded as uniformly distributed. As this is a single track bridge the live load is divided by two, and for convenience the weights are reduced to tons.

On the axis AN, Plate III, and to a scale of 8 feet to an inch, are marked the positions of the wheel loads and the beginning, middle and end of a portion of the train 20 feet in length. Through these points indefinite verticals are drawn. On the load line hk, at the left of the plate, the wheel loads are laid off successively to a scale of 10 tons to an inch followed by 20 tons—the weight of the 20 linear feet of train. The pole o is chosen at a point above the middle of the load line, the pole distance being five times the depth of the girder, or 27.5 feet. It is not necessary to draw the rays from the pole, as the direction of each ray is determined by the pole and a point on the load line through which points the edge of the triangle is passed in the construction of the equilibrium polygon $A'B'FL'M'N'$. As the train load is not concentrated at its center but is uniformly distributed the required part $L'e'N'$ of the equilibrium polygon is a parabola tangent to $L'M'$ at L' and to $N'M'$ at N' (Art. 10). The construction is indicated on the diagram. The portion $A'B'$ of the polygon is a straight line parallel to ho and is produced as far as needed.

The left half of the girder is divided into five equal parts each 5.5 feet in length, and the sections numbered as shown. After erasing the lines of action of the wheel loads above the equilibrium polygon, a series of verticals are drawn 5.5 feet apart. The two verticals each marked *cc'* are 55 feet apart and *c'c'* is the closing side of the equilibrium polygon for the position *cc* of the girder. For this position the first driver stands at section 3 of the girder. The ordinate *Pd'* represents the flange stress at section 1 for this position of the load, the ordinate *Qe'* for section 2, and so on. The closing lines *a'a'* to *g'g'* are drawn, and all points on these lines distant one space from the left end are united by the curve *PP*, those distant two spaces from the left end by the curve *QQ*, those distant three spaces by *RR*, those distant four spaces by *SS*, and those distant five spaces by the curve *TT*. The ordinates between the polygon and the curve *PP* indicate the successive values of the flange stress at section 1 as the girder is moved from left to right with respect to the load, or, in other words, as the live load passes over the girder from right to left. The maximum ordinate between these lines is directly over the first driver, indicating that when the load is placed so that the first driver stands at section 1 it will give the maximum flange stress at that section. The maximum is always located at a vertex of the polygon, that is, at one of the loads. When the ordinate is not at an intersection through which the upper curve was drawn its length should be tested by drawing the closing line for the required position of the load. The pole distance *H* being five times the depth of the girder this ordinate must be measured with a scale five times that used on the load line or 50 tons to an inch. Applying the scale it is found to be 38.0 tons. The maximum ordinates to *QQ* and *RR* are 66.2 and 85.9 tons respectively, both being at the second driver; and the maximum ordinates to *SS* and *TT* are 97.0 and 99.6 tons, both being at the third driver.

The center of gravity of the locomotive and tender is 3

feet behind the third driver. When the load is so placed that the center of the girder is midway between these two points, only these loads being on the girder, the absolute maximum flange stress will occur in the section at the third driver. The section is therefore 1.5 feet from the center of the girder and the flange stress is 100.3 tons. The closing line for this position is shown near $c'c'$.

On an axis equal to the span in length and divided like the girder into ten parts the flange stresses just found are laid off as ordinates and a curve drawn through their upper extremities. Only one half is shown in the lower left side of the plate, the other half being symmetrical with it.

The diagram for flange stresses due to the dead load is now constructed below this axis by the method of Art. 35, and by measuring the ordinates the flange stress for the different sections of one-half of the girder are found to be 6.3, 11.2, 14.7, 16.8, and 17.5 tons respectively. These are to be added to the live load stresses in order to obtain the maximum stresses in the flanges.

To determine the maximum live load shears another equilibrium polygon $A''B''D''L''N''$ is drawn by taking a new pole distance equal to the span, and as it is convenient to have the first side $A''B''$ horizontal the pole o' is placed directly opposite h, the beginning of the load line. From the first driver at C'' the span is laid off toward the right extending to f and then successive positions ee, dd, etc., of the girder are marked when it is moved toward the left 5.5 feet at a time. The ordinates $f''z$ . . $d''d$ . . $a''a$ are the corresponding reactions of the left support (Art. 37) and those portions of the ordinates above the line uwz are the maximum shears at the sections. The line uw is parallel to ae, ua equals 4 tons—the load on the pilot wheel—and wz is a part of $B''C''$ produced, w being 55 feet distant from B''. It is found that the maximum shear is caused in each section when the leading driver stands at that section and the load covers the right segment of the girder. For instance, when the first driver is at section 5 the shear is $a''u$ or 13.6 tons, when the pilot is placed at 5 the shear, measured on the ordinate at the left of $a''u$, is 10.6 tons, and for the second driver at 5 the corresponding ordinate (not shown on the plate) is still smaller. When the first driver is at the left support the reaction and also the shear at the support are equal to $f''z$ or 45.0 tons, $f''z$ lying between the polygon and the second side $B''C''$ produced, since the pilot is beyond the girder (Arts. 7 and 35). The shears for the sections from 0 to 5 are 45.0, 38.0, 31.3, 24.9, 18.9, and 13.6 tons respectively. If shears are desired for intermediate sections they may be measured directly on the diagram.

The above shears may also be obtained from the moment diagram used for finding the flange stresses. When the first driver is at section 5 the closing line of the polygon is $a'a'$ and, drawing parallel to this a ray through o, it is found to cut off on the load line a reaction of 17.6 tons or a shear of 13.6 tons. The other rays shown are parallel to the closing lines $b'b'$ to $f'f$. If one of the series of equidistant verticals did not coincide with the first driver another series of closing lines would have to be drawn to find the shears in this manner. The preceding method is usually the most economical in labor, but in this example the method just given has the advantage.

The shears due to dead load for the left half of the girder are given by the triangular shear diagram on the right of the lower end of the load line, and are to be added to the live load shears. Their values for the sections 0 to 5 are 7.0, 5.6, 4.2, 2.8, 1.4, and 0.0 tons.

Sometimes an approximate method is employed in which the pilot wheel is omitted from the system of loads. The differences between the true and the approximate shears in this

example are indicated on the diagram by the ordinates between the lines *uw* and $C''w$.

In practice the equidistant verticals corresponding to the divisions of the girder are drawn upon a separate sheet of tracing paper that may be shifted horizontally over the one on which the equilibrium polygons are constructed. This facilitates some parts of the process very materially.

If two coupled locomotives be specified instead of one the maximum flange stresses will be somewhat greater, as in that case the drivers of the second locomotive when placed at the various sections give the positions for maximum moment. This may be seen from the fact that the straight line $B'A'$ will be replaced by a broken line starting from B' and bending upward, thus raising the left ends of a number of closing lines and consequently increasing the ordinates between them and the polygon.

Prob. 52. A plate girder for a single track bridge has a span of 31 feet 6 inches and a depth of four feet. Find the flange stresses and shears due to two coupled Pennsylvania consolidation locomotives.

ART. 41. ANALYSIS OF A PRATT TRUSS.

Let the example whose computations were made in Part I, Art. 63, be taken in order to compare the accuracy of the graphic method with the analytic method. The truss is a through Pratt for a double track railroad, of 140 feet span, having 7 panels, each 20 feet long and 24 feet deep. The dead load per linear foot is 1 400 pounds or 0.7 tons, and the live load a Pennsylvania typical passenger locomotive and tender followed by a uniform train load of 3 000 pounds or 1.5 tons per linear foot. Of the dead load 400 pounds per linear foot is to be taken on the upper chord.

As the construction required by this example (see Plate IV) [illegible] ance only. The inner [illegible] and [illegible] distance is [illegible] length [illegible] through the [illegible] ordinates are [illegible] load line [illegible] the diagram [illegible] for the sec- [illegible] that for the [illegible] stresses.

[illegible] used instead [illegible] ordinates for the [illegible] as to place [illegible] right of these [illegible] the equilibrium [illegible]

[illegible] ordinates at d'', b'', [illegible] support for various [illegible] l is 140 feet, the [illegible] reaction when the [illegible] which measures (by [illegible]. The shear in the [illegible] portion of the load on [illegible] to the floor beam at [illegible] downward and drawing [illegible] the stress for that member [illegible] other load be placed at 2 a [illegible]

[illegible] the positive shear is a maximum [illegible] is at the panel point on the right. [illegible]

is in many respects similar to that in the preceding article, only those features which are different need explanation. The linear scale used on the original drawing was 10 feet to an inch, and the scale of force 20 tons to an inch. The pole distance is taken as twice the depth of the truss or 48 feet. The verticals above the equilibrium polygon are 10 feet or half a panel length apart, and the maximum ordinates for the sections through the panel points are drawn in heavy lines. The ordinates are measured by double the scale used in laying off the load line or 40 tons to an inch, and their values marked on the diagram. The maximum ordinate below the curve *ss*, drawn for the section through the fourth panel point, is smaller than that for the third, hence that is the limit required for the chord stresses.

If the Pennsylvania consolidated locomotive be used instead of the passenger locomotive, the maximum ordinates for the sections at 3 and 4 will be found at such positions as to place the engine beyond the bridge, and toward the right of these ordinates the stress curves will be parallel to the equilibrium polygon.

In the shear diagram on Plate IV the ordinates at d'', b'', etc., represent the reactions of the left support for various positions of the load since the pole distance $o'h$ is 140 feet, the length of the span. For instance, the left reaction when the first driver is at 2 is the ordinate below b'' which measures (by the scale of 20 tons to an inch) 79.2 tons. The shear in the second panel is this reaction minus that portion of the load on both pilot wheels carried by the stringers to the floor beam at 1, or 9.4 tons. Laying this off from b'' downward and drawing a line parallel to the diagonal *CD* the stress for that member is found to be 90.8 tons. If any other load be placed at 2 a smaller stress is found.

In the fifth and sixth panels the positive shear is a maximum when the second pilot wheel is at the panel point on the right.

The difference between the reaction and the shear is then 2.2 tons. The construction for both positions of the load is given for the fourth and fifth panels.

The tension in the suspender *BC* is equal to that portion of the loads between 0 and 2 that is carried by the stringers to the floor beam at 1, such loads being brought on as to make it a maximum. This condition requires the drivers to be near 1. To determine this the small equilibrium polygon is drawn directly below the locomotive wheels using a pole distance equal to the panel length. The pole is o''. When the first six wheels are so placed between 0 and 2 that their center of gravity is at 1, the reactions at 0 and 2 are each 16.7 tons, as given by the ordinate above the intersection of the outer sides produced (Arts. 7 and 37). The reaction at 1 is therefore $(8 + 8 + 20 - 16.7)2 = 38.6$ tons. The first driver is next placed at 1, thus moving the second tender wheel beyond 2, and the sides of the polygon are produced to meet the vertical under this load. The reaction at 1 is then $64 - (9.4 + 15.0) = 39.6$ tons. When the second driver is placed at 1 the reaction at 1 is $64 - (9.6 + 14.8) = 39.6$ tons. The greatest live-load tension in *BC* is therefore 39.6 tons.

It will be observed by the student that the ordinate of 9.4 tons under the first driver gives the amount that is laid off on five of the ordinates on the large shear diagram, while the small ordinate of 2.2 tons under the second pilot is deducted from three of them.

Another method of finding the stress in *BC* when the first driver is at 1 is shown on the diagram including the load line, in which *ot* is parallel to the closing line $f'd'$ and *ou* is parallel to the closing line $f'h'$.

The stress diagram for dead load when the diagonals all incline one way is shown on the left of the shear diagram. The initial tension in the counters of the third, fourth, and fifth panels is taken at 5.0 tons. The diagram is not shown, only a triangle being actually required.

The stresses thus obtained are for the chords,

	AD	*AF* and *Ah*	*RB* and *RC*	*RE*	*RG*
Dead load,	− 58.3	− 70.0	+ 35.0	+ 58.3	+ 70.0
Initial tension,	0	− 3.2	0	− 3.2	− 3.2
Live load,	− 130.8	− 154.5	+ 81.8	+ 130.8	+ 154.5
Maximum,	− 189.1	− 227.7	+ 116.8	+ 185.9	+ 221.3
Minimum,	− 58.3	− 73.2	+ 35.0	+ 55.1	+ 66.8

for the diagonals,

	AB	*CD*	*EF*	*GH*	*JK*	*LM*
Dead load,	− 54.7	+ 36.5	+ 18.2	0	− 18.2	− 36.5
Initial tension,	0	0	+ 5.0	+ 5.0	+ 5.0	0
Live load on right,	− 127.8	+ 90.8	+ 59.4	+ 33.5	+ 15.2	+ 2.9
Live load on left,	0	− 2.9	− 15.2	− 33.5	− 59.4	− 90.8
Final maximum,	− 182.5	+ 127.3	+ 82.6	+ 38.5	+ 2.0	0
Final minimum,	− 54.7	+ 33.6	+ 8.0	0	0	0

and for the verticals,

	BC	*DE*	*FG*	*HJ*	*KL*
Dead load,	+ 10.0	− 18.0	− 4.0	+ 10 0	+ 24.0
Initial tension,	0	− 3.9	− 7.7	− 7.7	− 3.9
Live load,	+ 39.6	− 45.6	− 25.8	− 11.7	− 2.3
Final maximum,	+ 49 6	− 67.5	− 37.5	—	—
Final minimum,	+ 10.0	− 21.9	− 11.7	—	—

The final maximum and minimum stresses are found in the manner given in Art. 30. As neither the wind stresses nor the initial tension in the lateral systems were determined in this example, the results for the chords are not the final maximum and minimum stresses.

On comparing these results with those obtained by the analytic method it is seen that the average difference between them is about 0.1 ton, the only difference exceeding this amount being 0.3 ton for the chord members *AD* and *RE*, the value obtained by computation being 131.1 tons. The analytic

method, however, gives another position of the live load which also satisfies the condition for the greatest stress in these chord members, namely, when the second driver stands at 2. The computed stress for the position is 130.3 tons. Two computations are also required for the stresses in $RB = RC$ and in GH.

The stresses in unsymmetrical trusses are as readily determined by the graphic method as for trusses with equal panels, while in the analytic method the numerical work of computation is materially increased.

Prob. 53. A through Pratt truss for a single track railroad bridge has 7 panels each 21 feet 6 inches long and 26 feet deep. The trusses are 14 feet 8 inches apart center to center of chords. The dead panel load is 1.93 tons on the upper and 6.33 tons on the lower chord. The live load is to consist of two coupled Lehigh Valley consolidation locomotives followed by a uniform load of 4 000 pounds per linear foot. Find the maximum and minimum stresses.

ART. 42. DOUBLE AND QUADRUPLE SYSTEMS.

For trusses having more than one system of webbing it is assumed that each system is affected only by the loads which it carries.

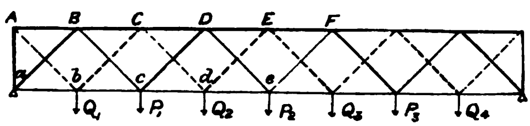


Fig. 88.

In the double system Warren truss in Fig. 88 the loads P_1, P_2, and P_3 are carried by the full line diagonals and the loads, Q_1, Q_2, Q_3, and Q_4 by the diagonals drawn in broken lines. The truss is therefore regarded as composed of two separate trusses having common chords. The stresses in each system may then be determined and the results combined. In this case, however, either the dead load stresses in all the members or the live load stresses in the chords may be found by means of one diagram. The reaction at the left support is in equilibrium with the stresses in Aa, Ba, and ab, but the compression in Aa is known since it equals the reaction due only to the loads Q, thus leaving only two unknown stresses. The stress diagrams may therefore be readily constructed. The maximum live load stresses in the diagonals are obtained by considering each system separately.

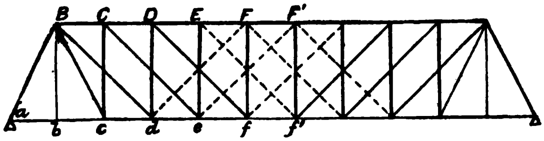


Fig. 89.

For the Whipple truss in Fig. 89 both dead and live load stresses must be found for each system. For dead load the division is to be made into two systems as shown in Fig. 90. The stress in CD is then the sum of the stresses in BD and CE. The same arrangement is required for the chord stresses, which are due to a uniform live load. For the live load stresses the systems are divided as in Fig. 91, all the diagonals sloping one way. Only the loads supported by one of the systems are considered in finding the live stress in any web member of that system. The method employed is that of Art. 30, the labor of tabulation being materially lessened by noting the general statements made in that article in regard to the maximum and minimum stresses in web members. The Whipple is a double intersection Pratt truss.

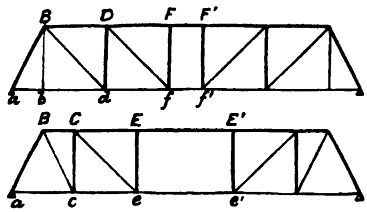


Fig. 90.

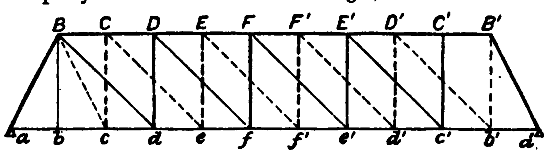


Fig. 91.

When one excess load is used in connection with uniform panel live loads, it must be placed on that system which gives

the greatest chord stresses; and for two excess loads the first may be on one system and the second on the other depending upon their distance apart and the panel length. For maximum stresses in the webbing the excess loads are always placed at the head of the train.

If concentrated wheel loads are to be employed it will be best to always place the first driver at the panel point. Each system is regarded as acting independently and as being strained only by the loads transferred to it by the stringers and floor beams. As part of the weight of the pilot is carried by the stringer to the other system, that part is disregarded in obtaining the stresses in the bracing. For the chord stresses the locomotives are kept near the middle of the truss and the weights transferred to each system are used only in determining the chord stresses for that system.

A quadruple Warren truss or lattice girder is treated in a similar manner to the double system Warren, and also requires but one diagram for dead load.

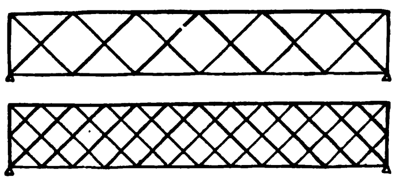


Fig. 92.

Prob. 54. A double system deck Warren truss of 100 feet span has 10 panels and is 10 feet deep. The dead load per linear foot per truss is 560 pounds, and the train load 1 800 pounds, which is preceded by two heavy locomotive panel loads of 65 000 pounds each. Find the maximum and minimum stresses in all the members.

APPENDIX.

ANSWERS TO PROBLEMS.

Prob. 1. 56.8 pounds, making an angle of 37° 35′ with the smaller force.

Prob. 2. 43.6 pounds, 36° 35′ and 83° 25′.

Prob. 4. 107.8 pounds and 129° 20′.

Prob. 5. 2 800 pounds.

Prob. 6. $S_2 = + 5.56$, $S_3 = - 3.27$, $S_4 = - 5.86$, and $S_5 = + 5.95$ tons.

Prob. 8. $S_1 = - 43.6$, $S_2 = S_4 = + 109.1$, and $S_3 = + 43.6$ pounds.

Prob. 9. Resultant = 279.7 pounds, and angle with greater force = 1° 45′.

Prob. 11. Resultant = 4 tons, is parallel to forces and 6 feet from greater force.

Prob. 12. 177.1 and 192.9 pounds.

Prob. 13. Maximum shear = ± 4 000 pounds, maximum moment = + 20 000 pounds-feet.

Prob. 15. Maximum shear = − 6 tons, maximum moment = − 30 tons-feet.

Prob. 16. Maximum shear = − 6 tons, maximum moment = − 30 tons-feet.

Prob. 17. 0.70 inches from the back of channel iron.

Prob. 18. 500 pounds, 15.27 feet from the first force.

Prob. 19. $I = A \times A' = 7.10 \times 10.81 = 76.75$ inches^4, and $I' = A \times A'' = 7.10 \times 0.51 = 3.62$ inches^4.

Prob. 22. Stress in $AC = BC = -2\,290$ pounds, and in $CD = +2\,050$ pounds.

Prob. 23. Apex loads = 1.48, 1.20 tons; reactions = 4.44, 3.60 tons.

Prob. 24. 2.79, 8.37, 2.04, and 6.12 tons.

Prob. 25. Apex loads = 2.15 and 1.61 tons. Dead load stresses are: in upper chord, − 12.03, − 9.78, − 10.10; in lower chord, + 10.75, + 6.45; and in braces, − 2.30, − 2.30, + 4.29 tons. The corresponding snow load stresses are, − 9.00, − 7.32, − 7.56; + 8.05, + 4.83; − 1.72, − 1.72, + 3.21 tons.

Prob. 27. $AB = 2\,035$, $BC = 2\,815$, and $CD = 855$ pounds, the normal wind pressures being 38.2 and 15.6 pounds per square foot.

Prob. 28. Apex load = 1.86 tons; reactions = 3.84 and 1.74 tons. Stresses in upper chord, − 5.82, − 6.75, − 5.36, − 3.49, − 3.49, − 3.49; in lower chord, + 6.51, + 4.43, + 2.35, + 2.35, + 2.35; and in braces, − 2.08, + 2.94, − 3.12, + 3.75, 0, 0, 0, and 0 tons.

Prob. 32. Apex load = 2.05 tons; reactions at free end, 2.30 and 4.99 tons. Stresses for wind on fixed side are: in upper chord, − 8.93, − 7.45, − 5.98, − 4.51, − 5.04; in lower chord, + 11.22, + 8.98, + 6.73, + 4.48; and in braces, − 2.52, + 1.15, − 3.21, + 2.30, − 4.11, + 3.45, and 0 tons. Lower chord stresses are diminished 3.75 tons for wind on free side.

Prob. 34. Dead apex loads = 0.70, 1.40, 1.40, etc.; snow apex loads = 0.43, 1.08, 1.30, etc.; and wind apex loads = 1.74, 2.35, and 0.70 tons. Maximum stresses in upper chord, − 11.0, − 10.3, − 8.7, − 9.4; in lower chord, + 9.4, + 8.4, + 6.4; and in braces, + 1.9, + 2.7, + 1.7, + 3.3 tons. Minimum stresses, − 3.9, − 3.3, − 3.4, − 4.0; − 0.7, + 0.8, + 2.3; + 1.0, − 1.2, − 0.2, and − 0.5 tons.

Prob. 36. Dead, snow, and wind apex loads are 0.91, 0.84, and 1.49 tons. Maximum stresses in upper chord, − 14.45,

— 11.38, — 9.56; in lower chord, + 13.96, + 9.41; and in braces, — 3.82, + 2.06, — 3.21, + 3.44 tons. Minimum stresses, — 5.09, — 4.07, — 3.39; + 4.55, + 3.34; — 1.02, + 0.55, — 0.85, and + 1.21 tons.

Prob. 37. $cd = ce = -4.31$, $cg = ch = -1.73$, $ad = +3.91$, $bf = +1.26$, $bh = +1.57$, $de = 0$, $ef = +3.55$, $fg = +0.39$, and $gh = 0$ tons.

Prob. 38. Apex loads = 0.6, 1.2, and 0.6 tons. Designating the members as in Fig. 51, the stress $bf = -4.15$, $cg = -3.61$, $dk = dl = -2.06$, $ef = +3.76$, $eh = +1.50$, $el = +1.88$, $fg = -1.08$, $gh = +2.36$, $hk = +0.47$, and $kl = 0$ tons.

Prob. 39. Dead panel load per truss = 1.98, live = 4.70 tons.

Prob. 40. Panel loads = 2.83 on upper, and 5.67 tons on lower chord. Stresses in upper chord, — 39.4, — 47.3, — 47.3; in lower chord, + 23.6, + 23.6, + 39.4, + 47.3; and in web members, — 34.7, + 5.7, + 23.2, — 11.3, + 11.6, 0, and 0 tons.

Prob. 41. Maximum stresses in upper chord, — 46.6, — 123.3, — 169.2, — 184.5; in lower chord, + 92.0, + 153.3, + 183.9 tons. Minimum stresses, — 12.2, — 31.5, — 43.0, — 46.8; + 23.1, + 38.5, + 46.2.

Prob. 43. Panel load = 0.79 tons. Stresses in the chords, 3.4, 6.2, 8.4, 9.9, 10.9, 11.2; in diagonals, — 5.5, + 4.5, + 3.5, + 2.5, + 1.5, + 0.5; and in verticals, + 0.8, — 2.8, — 2.0, — 1.2, — 0.4, and 0 tons.

Prob. 44. Panel load due to truss is 1.074, and that due to train is 4.0 tons. Stresses in upper chord for south wind, 0, + 2.0; for north wind, — 2.0, — 2.7 tons. In the lower chord, 0, + 15.9, + 25.4; — 15.9, — 25.4, — 28.5 tons. Maximum wind stresses in diagonals of upper lateral system, + 2.6, + 0.9; in struts, — 1.6, — 1.1 tons. In lower lateral system, + 20.3, + 13.3, + 7.3; — 12.7, — 8.3, and — 5.1 tons.

Prob. 46. The greatest reduction of stress is in *RC*, and equals $4.7 \div 76.4 = 6.2$ per cent.

Prob. 47. Maximum stresses in upper chord, — 56.0, — 53.2,

— 52.1; in lower chord, + 50.0, + 50.0, + 51.3; in main ties, + 8.3, + 8.7; in counter ties, + 7.3, + 8.0; and in verticals, + 10.0, + 9.4, + 9.0 tons. Minimum stresses in chord, — 14.0, — 13.3, — 13.0, + 12.5, + 12.5, + 12.8; in diagonals, 0; and in verticals, + 2.5, + 0.2, — 0.3 tons.

Prob. 48. Maximum stresses in the chords, — 100.0, + 109.7, + 105.9, + 103.0, + 101.1, + 100.1; in main ties, + 9.0, + 10.2, + 10.9; in counters, + 7.7, + 9.0, + 10.2, and in verticals, — 10.0, — 12.4, — 14.2, — 15.2 tons. Minimum stresses in the chords, — 30.0, + 32.9, + 31.8, + 30.9, + 30.3, + 30.0; in diagonals, 0; and in verticals, — 3.0 tons.

Prob. 51. 42.5 tons.

Prob. 52. Dividing the span into eight equal parts, the flange stresses at the sections are 0.0, 16.0, 26.4, 32.6, and 35.1 tons. The absolute maximum is 35.2 tons at 4 inches from center of girder. The shears are 20.1, 16.3, 12.8, 9.4, and 6.1 tons.

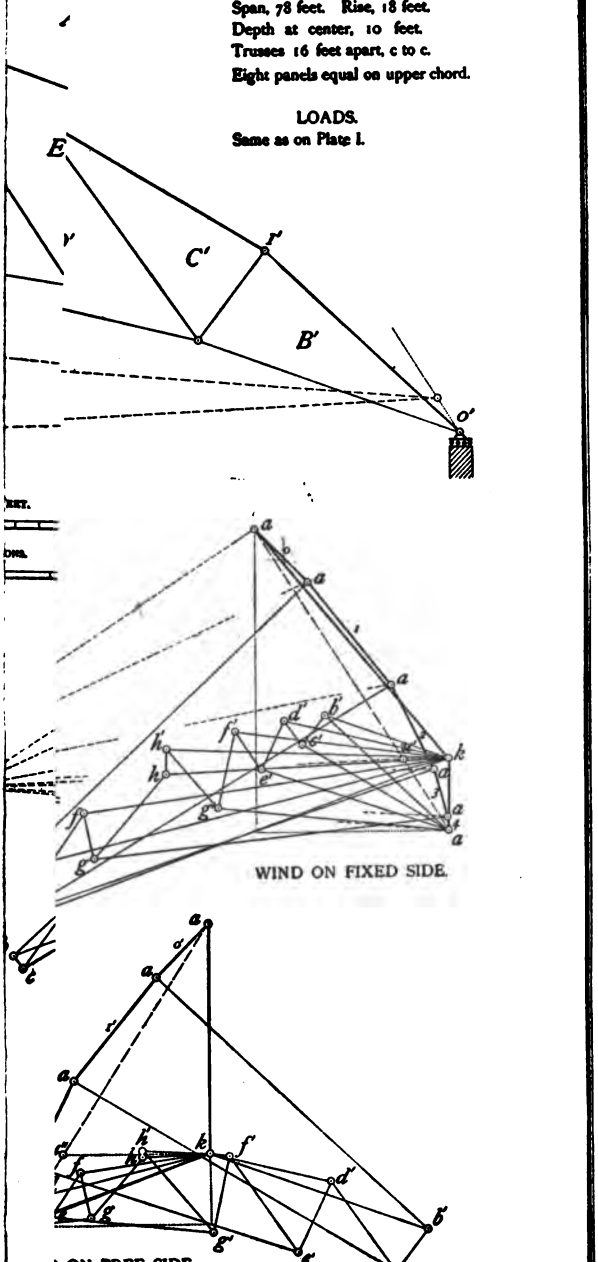
Prob. 53. Maximum stresses in chords, 86.2, 138.1, 167.1; in end post, — 135.2; in main ties, + 95.4, + 60.1, + 28.0; in counter ties, + 3.5, + 28.0; and in verticals, + 40.8, — 48.3, and — 23.5 tons. Minimum stresses in the chords, 20.5, 34.2, 41.0; in end posts, — 32.2; in main ties, + 17.9, 0, 0; in counter ties, 0, 0; and in verticals, + 6.3, — 1.9, and — 1.9 tons.

DIMENSIONS.

Span, 78 feet. Rise, 18 feet.
Depth at center, 10 feet.
Trusses 16 feet apart, c to c.
Eight panels equal on upper chord.

LOADS.

Same as on Plate I.



H.S.J.

PLATE GIRDER ;OADS.

; AND LOADS.

ingle track railroad.

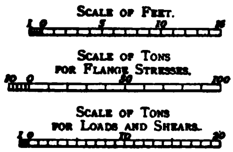
Depth, 5.5 feet.

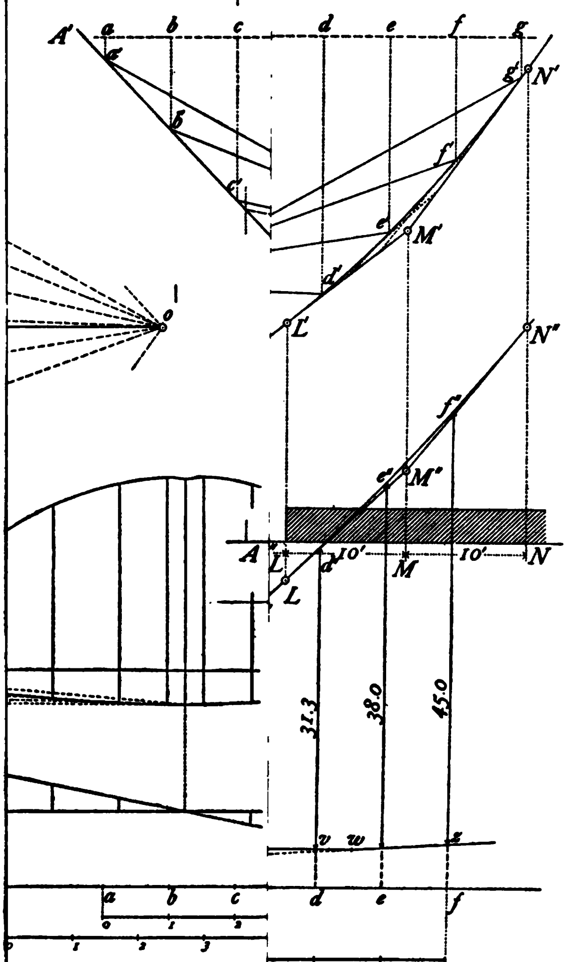
girders, 16.45 tons.

pounds per lin. ft.

Valley consolidation

train as per diagram.





N.S.J.

Plate_IV.

TT TRUSS UNDER LOC